Novel Systematic Errors in Microwave Space Clocks and Optical-Lattice Clocks

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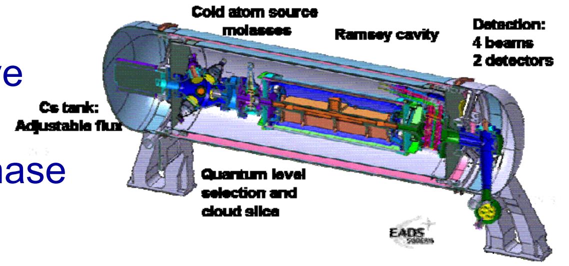
- Accuracy Evaluation of PHARAO
  - Microwave Lensing (aka Microwave photon recoil) frequency shift.
  - 1st order Doppler shifts – Cavity phase
  - Cold Collision shift
- Clock frequency shift from ultracold Fermion collision



On Earth: $\sigma_y < 4.10^{-13}\ t^{-1/2}$, $\sigma_B \sim 10^{-15}$
In space: $\sigma_y < 10^{-13}\ t^{-1/2}$, $\sigma_B \sim 10^{-16}$

PHARAO (10^{-16})	Shift	Uncertainty
Quadratic Zeeman	440	0.4
Blackbody radiation	-170	0.5
Ultracold collisions	-25	1.2
Cavity Phase	0.3	1
Microwave Lensing		1.4
Total		**2.2**

Phil Peterman, Kurt Gibble, Philippe Laurent, Christophe Salomon

Support from CNES, ESA,CNRS, NASA, LNE, SYRTE, ENS, Penn State, UPMC, & la Ville de Paris.

Is an atom's recoil equal to ħk?

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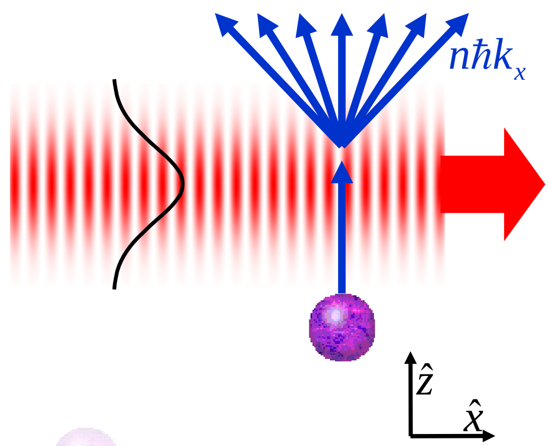
Finite beam:

Maxwell: $\left(\nabla^2 + k^2\right)E = 0$

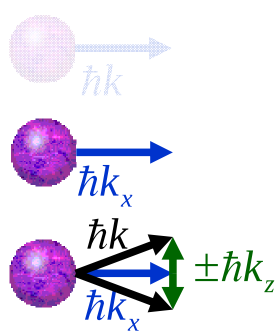
so $k_x^{\ 2} + k_y^{\ 2} + k_z^{\ 2} = k^2 \ \Rightarrow k_x < k$

$$k_{y,z} \approx \frac{2}{w_0 = 2mm} \Rightarrow \delta k_x = -8\,ppb$$



Three appealing choices:

1. The photon momentum comes in discrete units of k, in the x direction.
2. The atom has a recoil of k_x in the x direction; v_y & v_z are unchanged.
3. The atom has a recoil of k_x in the x direction, and also $\pm k_y$ & $\pm k_z$.

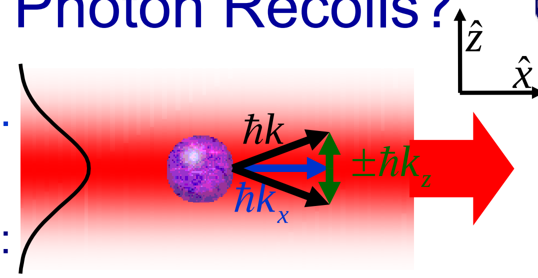


Wicht…, Chu, Phys. Script '02
Cladé…, Biraben, PRL '06
Gould, Ruff, Pritchard, PRL '86
Wicht, … Chu, PRA '05

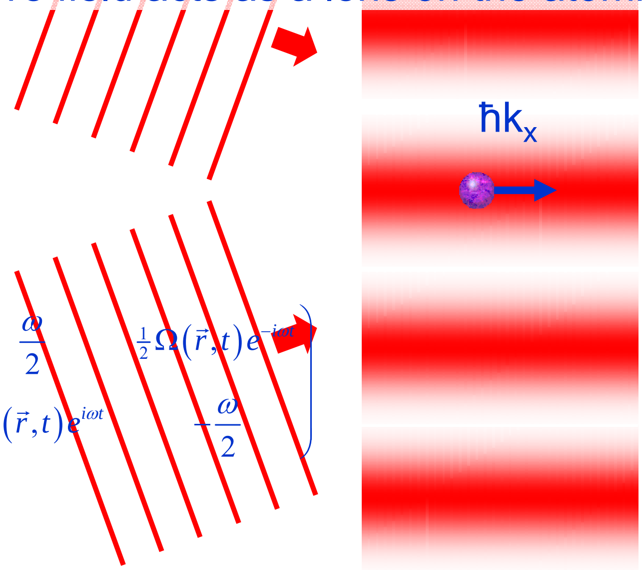
Transverse (Microwave) Photon Recoils?

There is no grating in the z direction.
→ No recoil in z direction.

"Microwave" Stern-Gerlach regime:



Same problem for microwave clocks: The dipole force of the microwave field acts as a lens on the atomic wavefunction.



$$H_{\text{int}} = \begin{pmatrix} \frac{\omega}{2} & \frac{1}{2}\Omega(\vec{r},t)e^{-i\omega t} \\ \frac{1}{2}\Omega(\vec{r},t)e^{i\omega t} & -\frac{\omega}{2} \end{pmatrix}$$

KG PRL '06

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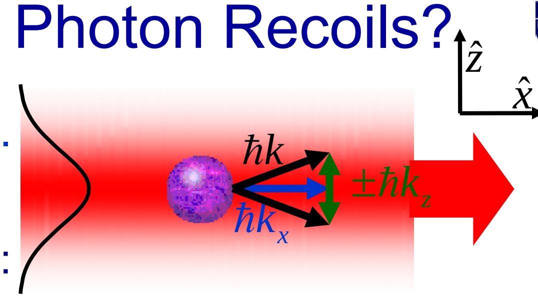

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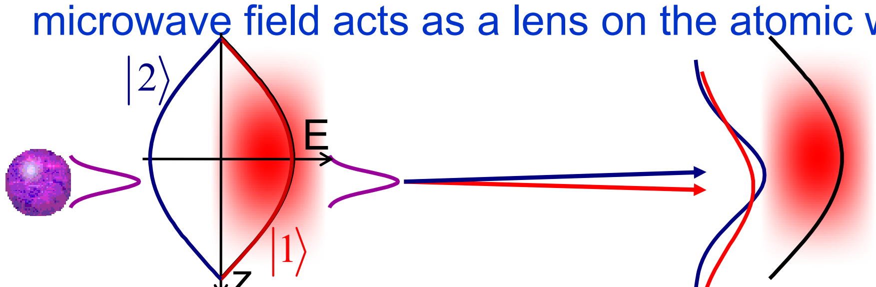
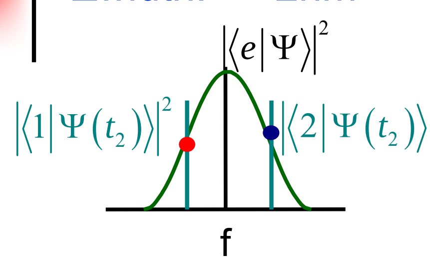


Shift: ≈ ±4nm

Δwidth: ≈ ±2nm



$$\delta P = \left[|\langle 2|\Psi(t_2)\rangle|^2 - |\langle 1|\Psi(t_2)\rangle|^2 \right] \sin\left[\frac{\pi}{2}\cos(k_x x) \right]$$

$$\delta\upsilon \neq \frac{\hbar k^2}{4\pi m} \qquad \delta\upsilon = \frac{\pi}{2}\Delta\upsilon_R \frac{w_1}{w_2}$$

$$\frac{\delta\upsilon}{\upsilon} \neq 1.5\times10^{-16} \qquad \frac{\delta\upsilon}{\upsilon} \approx 8\times10^{-17}$$

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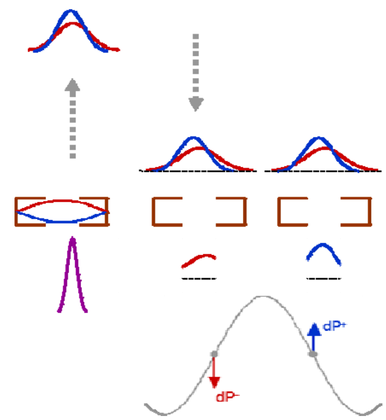
PHARAO Microwave Lensing

$$\delta P = \frac{1}{2N}\int\left[\left|\langle 2|\Psi\rangle\right|^2 - \left|\langle 1|\Psi\rangle\right|^2\right]\sin\left[\theta(r_2)\right]W_d(\vec{r}_d)\,dz_2 d\vec{r}_2$$



•For slow velocities & rectangular cavity: $\frac{\delta\nu}{\nu} = \frac{\pi}{2}\frac{t_1}{t_2}\frac{v_R}{v} \approx 1.2\times 10^{-16}$

•Analytic, with usually quite good approximations.

•Including selection aperture, final clock aperture, linear deflections (k^2), and uniform detection:

$$\frac{\delta\nu}{\nu} \approx 1.2 \text{ to } 1.3\times 10^{-16}$$

•With a full FEM calculation, uncertainty is trivially small, $<3\times 10^{-17}$.

•Include lower aperture, higher k.

(Need to add detection inhomogeneities & rounded corners of apertures.)

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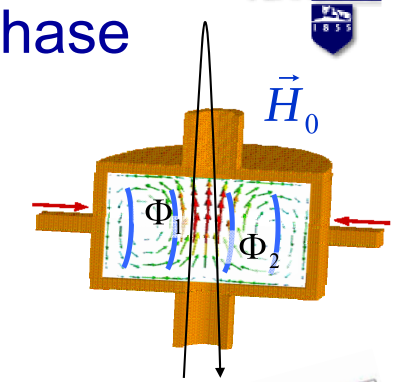
Ultracold collisions	-25	1.2
Cavity Phase	0.3	1
Microwave Lensing	1.2	0.3
Total		**1.7**

KG PRL '06
Li, KG, & Szymaniec Metrol. '11
Weyers, Gerginov, Nemitz, Li, KG, Metrol. '12

Doppler Shifts - Cavity Phase



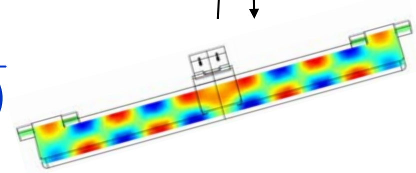
1st order Doppler shift: motion with spatial phase variation of an electromagnetic field.

– TE_{011} cylindrical cavity

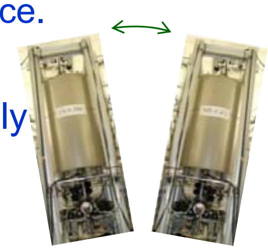
Standing wave + traveling wave
→ spatially varying phase

$$\delta\nu = \frac{\Phi_1 - \Phi_2}{\pi}\Delta\nu$$

$$\vec{H} = \vec{H}_0 + i\vec{g} \qquad \vec{E} = i\vec{E}_0 - \vec{f} \qquad \Phi = -\frac{g_z(\vec{r})}{H_{0z}(\vec{r})}$$



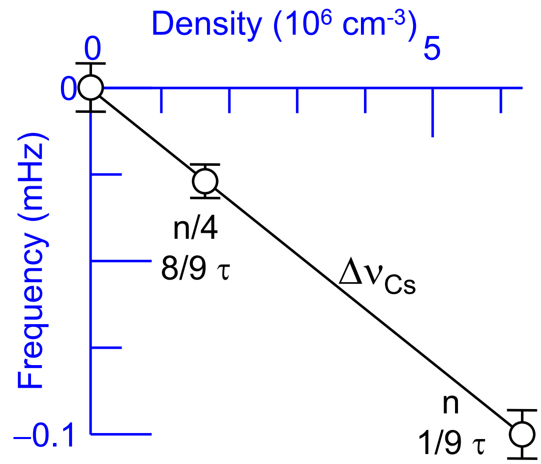
- Recent stringent test with fountains.
- PHARAO acts as a fountain fed from top endcap.
  – Large longitudinal phase gradient → power dependence.
- Large 3D FEM calculations,1 TB RAM
  – Dense, adaptive mesh, solve for $\mathbf{H_0}$ and **g** separately
- Calculate longitudinal phase shifts (m= 0 & 2).
- Measure phase gradients by tilting (m=1).

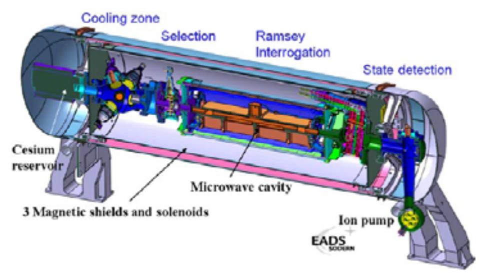


Li & KG, Metrologia '04 & '10, Li, KG, Szymaniec, Metrologia '11
Guéna, Li, KG, Bize, Clairon, PRL '11. Weyers, Gerginov, Nemitz, Li, KG, Metrol. '11

Ultracold Collision Shift & State Selection

- Collisions shift phase of clock coherence → frequency shift.
- Shot-noise-limited → operate at n & n/4.
- Need same density distribution for density extrapolation, DCP, & Microwave lensing.

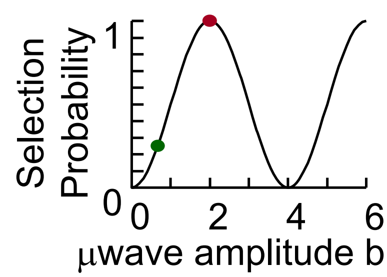


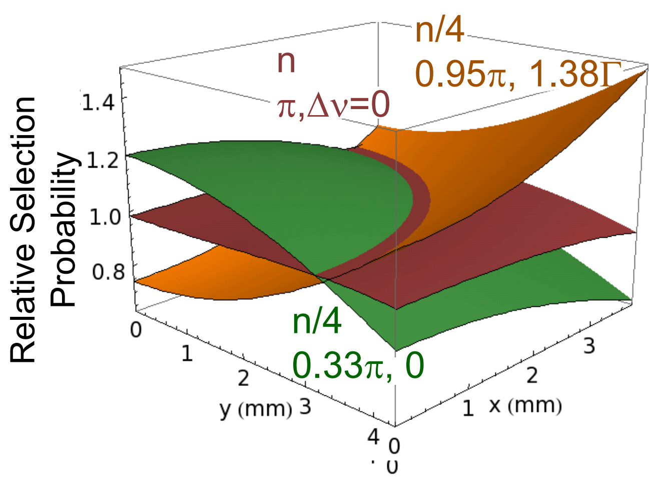


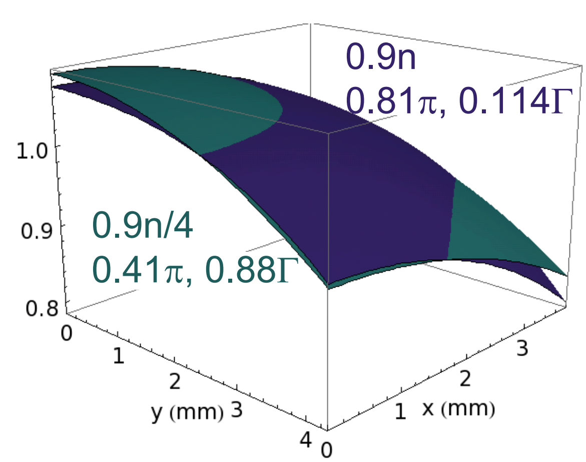
KG & Chu, PRL '93

Ultracold Collision Shift & State Selection

- Cylindrical cavity with rectangular aperture.
- FEM models of PHARAO Selection cavity.







- Need same density distribution for density extrapolation, DCP, & Microwave lensing.
- Selection using detuning and amplitude to make insensitive.

KG EFTF '12 Fertig & KG PRL '00

s-Wave Collisional Frequency Shift of a Fermion Clock (e.g. Lattice Clock)

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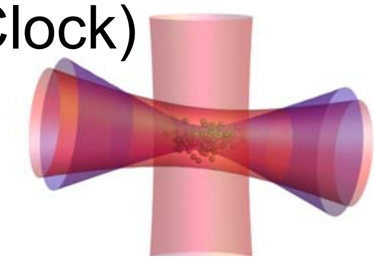
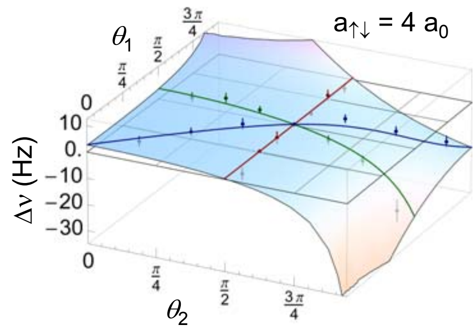
- Background
- Understanding the s-wave frequency shift of 2 fermions
- Experiment: ^{6}Li Fermion "Clock"
- Observation of distinguishing characteristics
- Tune scattering length near an s-wave Feshbach resonance
- Measuring $\Delta\Omega$
- Spin waves





Eric Hazlett, Yi Zhang, Ronald Stites, Kurt Gibble, & Ken O'Hara

Support from NSF, DARPA, Sloan, ONR, Penn State, & la Ville de Paris.

Ultracold Fermions (for Optical Lattice Clocks)

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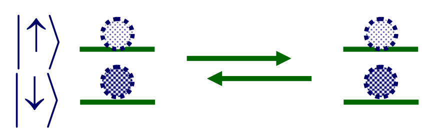
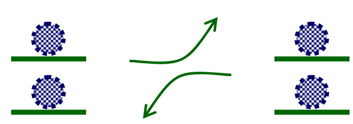
- At ultracold temperatures, only s-wave scattering.
- Antisymmetric wavefunction → no scattering of identical superpositions!

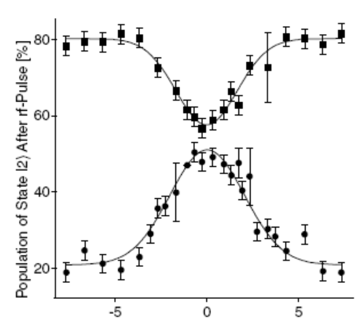
$\sigma = 0$



- Decoherence → distinguishable fermions → collisions

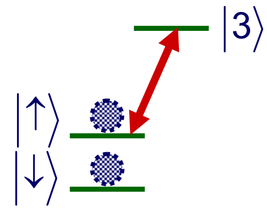
$\sigma \neq 0$ $\quad \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$ $\quad \frac{|\uparrow\rangle - |\downarrow\rangle}{\sqrt{2}}$





- No Shift! Despite being distinguishable, fermions act as if they're indistinguishable!
- $[H_{light}, V_{\uparrow\downarrow}]=0$ →

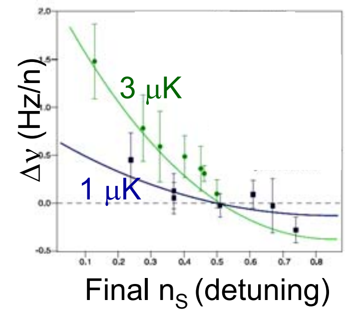
Fermions are Universally Immune to Collisions!



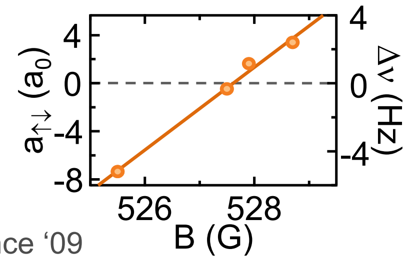
Zwierlein, Hadzibabic, Gupta, & Ketterle, PRL '03

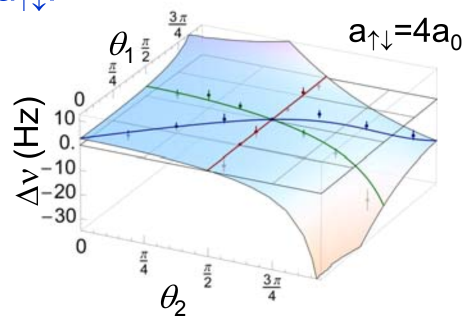
Fermion Clock Collision Shifts

- Collision shifts of ^{87}Sr & ^{171}Yb fermions.
  - Treated shift as $\Delta\nu \propto n_{\uparrow} - n_{\downarrow}$.
- $[H_{light}, V_{\uparrow\downarrow}] \neq 0$ for clock fields with spatial variations → s-wave shift is allowed.
- Unique behaviors of Fermion shift, as compared to bosons:
  1. Shift $\Delta\nu$ is independent of 1st Ramsey pulse area θ_1 ($n_{\uparrow} - n_{\downarrow}$).
  2. Depends strongly on 2nd Ramsey pulse θ_2.
     not observed for ^{87}Sr & ^{171}Yb → p-wave (same as usual boson s-wave).
  3. Proportional to s-wave scattering length $a_{\uparrow\downarrow}$.
  4. Increases with inhomogeneity as $\Delta\theta^2$.



^{6}Li s-wave Feshbach resonance





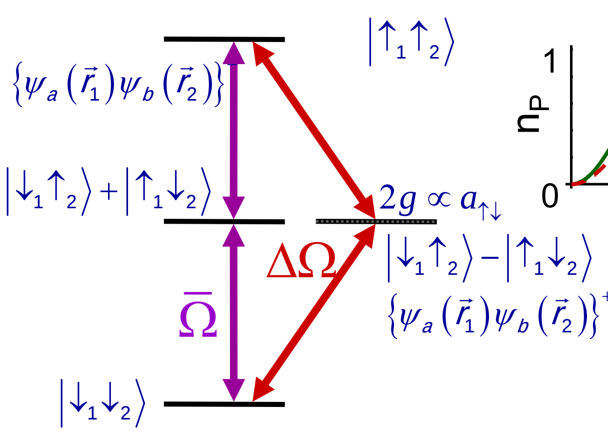
Campbell … Ye, Science '09
Lemke, … Oates, PRL '09
KG, PRL '09

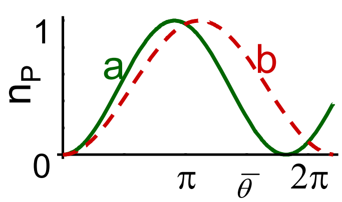
Maineult, Deutsch, KG, Reichel, Rosenbusch, PRL '12
Hazlett, Zhang, Stites, KG, O'Hara PRL '13

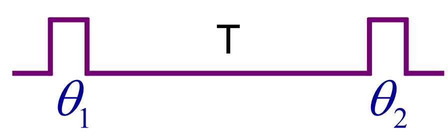
Fermion Clock Collision Shift

- Many particles are sum of pair-wise effects
- Basis – Singlet and Triplet states of 2 atoms:

$$\Omega = \Omega_a e^{i\Delta\omega_a t} \qquad \Omega' = \Omega_b e^{i\Delta\omega_b t}$$





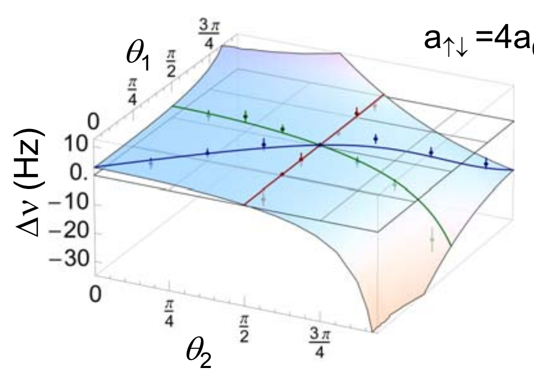


$$\Delta\nu = \frac{g}{2\pi}\frac{\sin(2\Delta\theta_1)\sin(\Delta\theta_2)\cos(\bar{\theta}_2)}{\sin(\bar{\theta}_1)\sin(\bar{\theta}_2)}$$

- $\Delta\theta_1 = 0 \Rightarrow s = 0$; identical fermions
  - $\Delta\nu$ is not proportional to $n_S - n_P$

$$\cos(\bar{\theta}_1) = \frac{n_S - n_P}{n}$$

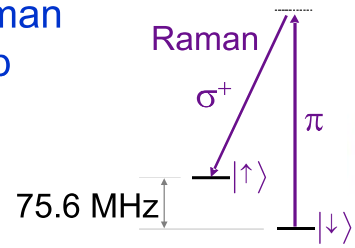
- No shift for $\Delta\theta_2 = 0$
- No shift for $\bar{\theta}_2 = \pi/2$

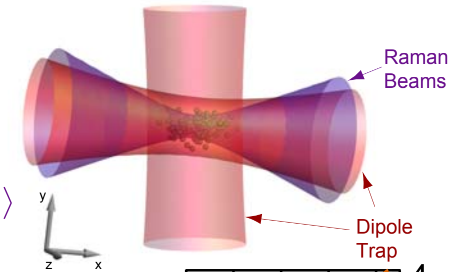


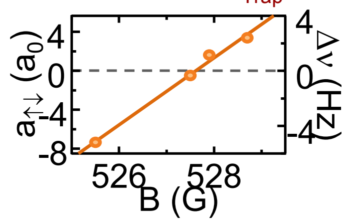
KG, PRL '09
Hazlett, Zhang, Stites, KG, O'Hara PRL '13

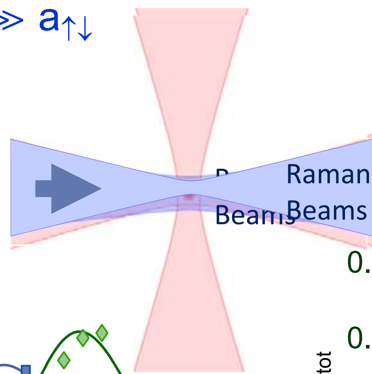
Ultracold Thermal ^{6}Li Fermion "Clock"

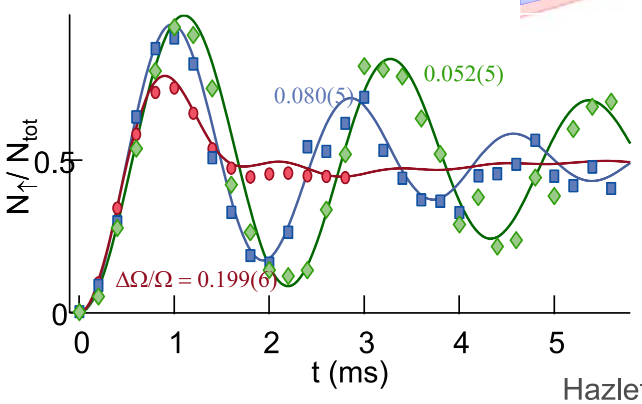
- Inhomogeneity from Raman beams – nuclear spin flip
- Resolved sideband: $\nu_{x,y,z}$={3.4, 7, 11} kHz
- Neglect trap-state changing collisions: $\lambda_{dB} \gg a_{\uparrow\downarrow}$

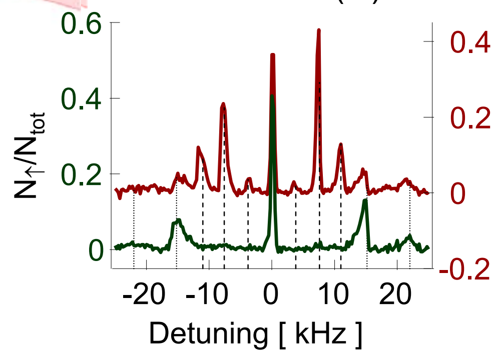








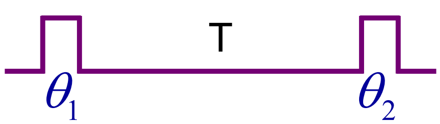




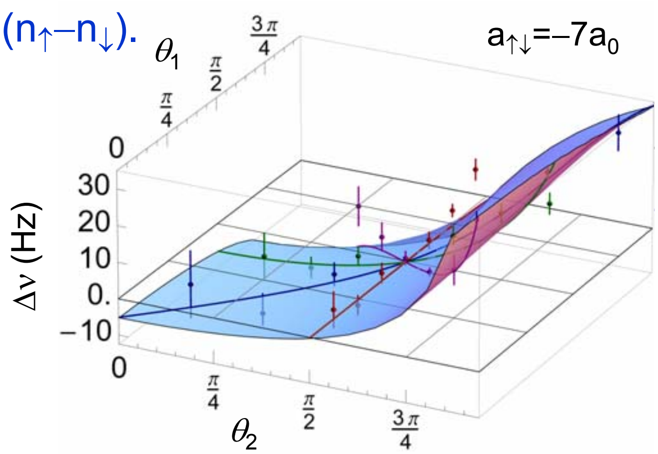
Hazlett, Zhang, Stites, KG, O'Hara PRL '13

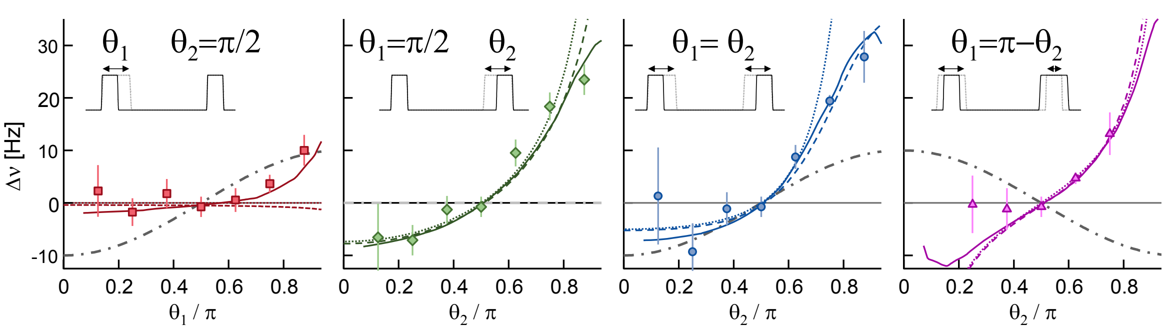
Fermion Clock Collision Shift

1. Shift $\Delta\nu$ is independent of θ_1 ($n_\uparrow - n_\downarrow$).
2. Depends strongly on θ_2.
3. Proportional to $a_{\uparrow\downarrow}$.
4. Increases as $\Delta\theta^2$ (next).



$$\Delta\nu = \frac{g}{2\pi}\frac{\sin(2\Delta\theta_1)\sin(\Delta\theta_2)\cos(\bar{\theta}_2)}{\sin(\bar{\theta}_1)\sin(\bar{\theta}_2)}$$

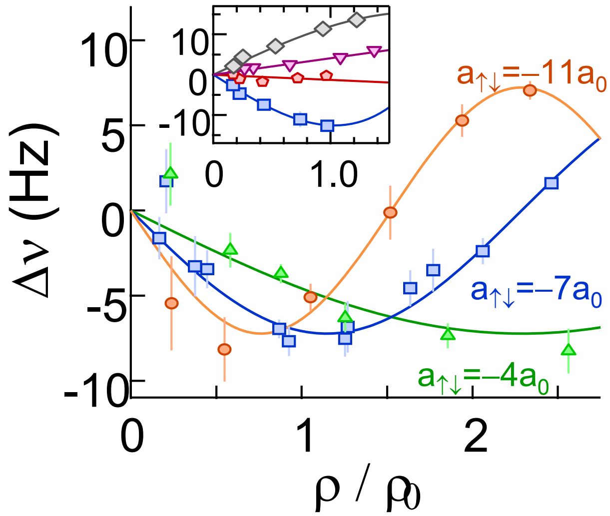




Hazlett, Zhang, Stites, KG, O'Hara PRL '13

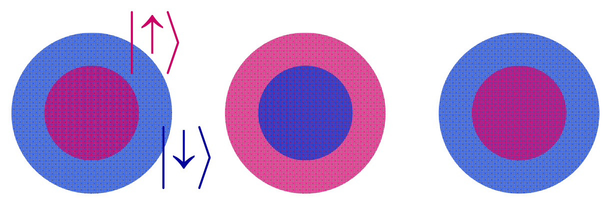
Spin Waves in Resolved Sideband Regime

- Fermion clock shift $\Leftrightarrow$ spin waves
  - Low vibrational states have high Ω
  - Phase of singlet state evolves through π, and 2π.



$$\Psi = se^{-i\omega_{ex}T}\left|0,0\right\rangle\left\{\left|\psi_a\psi_b\right\rangle\right\}^+ + \left[t\left|1,0\right\rangle + u\left|1,1\right\rangle + d\left|1,-1\right\rangle\right]\left\{\left|\psi_a\psi_b\right\rangle\right\}^-$$

$$= \tfrac{1}{\sqrt{2}}\left(t + se^{-i\omega_{ex}T}\right)\left\{\left|\uparrow\psi_a\right\rangle\left|\downarrow\psi_b\right\rangle\right\}^- + \tfrac{1}{\sqrt{2}}\left(t - se^{-i\omega_{ex}T}\right)\left\{\left|\downarrow\psi_a\right\rangle\left|\uparrow\psi_b\right\rangle\right\}^- + \left[u\left|\uparrow\uparrow\right\rangle + d\left|\downarrow\downarrow\right\rangle\right]\left\{\left|\psi_a\psi_b\right\rangle\right\}^-$$

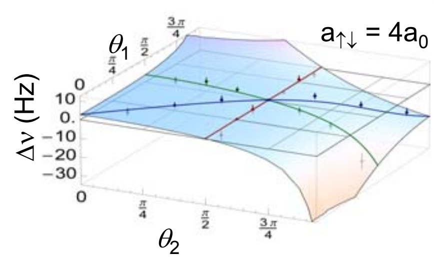


Maineult, Deutsch, KG, Reichel, Rosenbusch, PRL '12

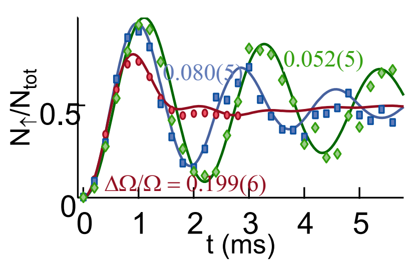
Hazlett, Zhang, Stites, KG, O'Hara PRL '13

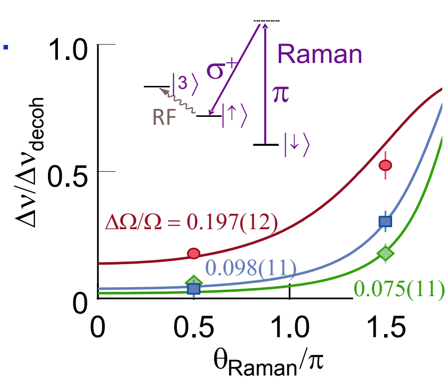
Frequency Shift versus $\Delta\Omega$

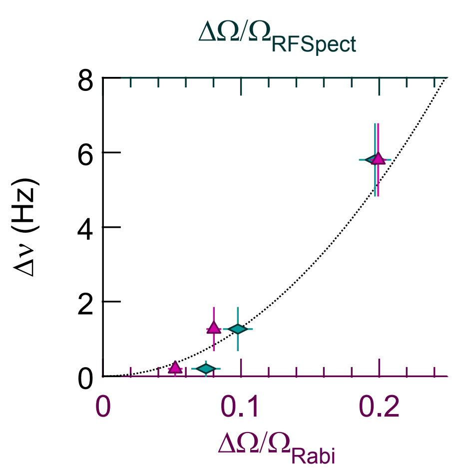
1. Shift $\Delta\nu$ is independent of θ_1 ($n_\uparrow - n_\downarrow$).
2. Depends strongly on θ_2.
3. Proportional to $a_{\uparrow\downarrow}$.
4. Increases as $\Delta\theta^2$.



Measure $\Delta\theta^2$ by Rabi flopping and frequency shift of $\uparrow$ to 3 transition.







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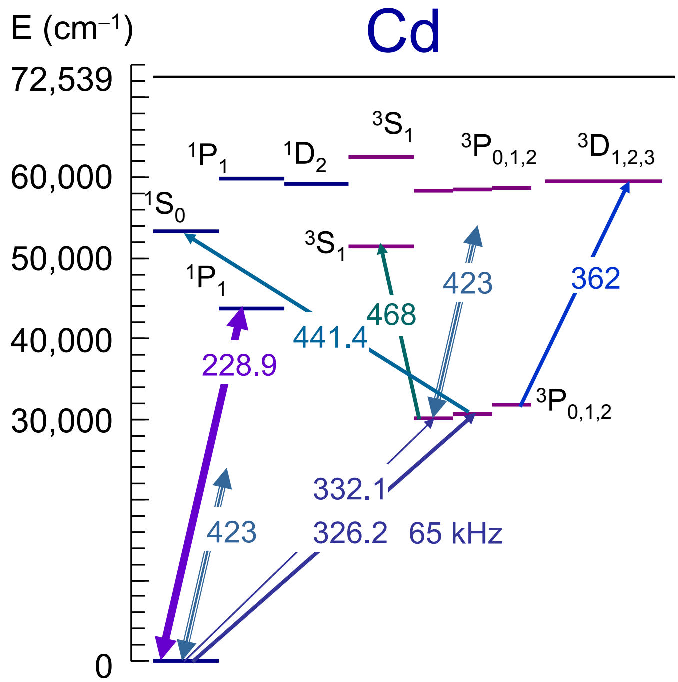
Microgravity Cd Lattice Clock

- Hg & Cd have small BBR shifts & similar level structures.
  - less repumping
- 2 fermionic spin ½ isotopes, ^{111}Cd and ^{113}Cd – selectability of ultracold collision scattering lengths.
  - Bosons: ^{110}Cd, ^{112}Cd, ^{114}Cd & ^{116}Cd.
- Wavelengths and sideband cooling are easier for Cd but $^3P_1$ MOT is harder than Hg.



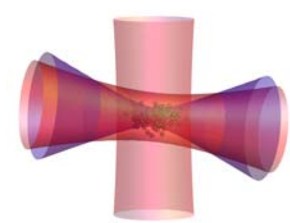
Binnewies, … Riehle, Helmcke, …. Rasel, Ertmer PRL '01

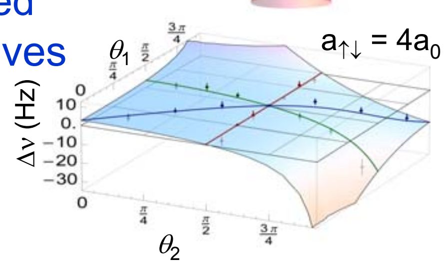
Curtis, Oates, & Hollberg, PRA '01 Friebe, … Ertmer … Rasel, New J. Phys. '11

Summary

- Progress on PHARAO accuracy evaluation
  - Microwave lensing uncertainty is negligible
- First observation of ultracold collisional frequency shift of a Fermi gas.
  1. Shift $\Delta\nu$ is independent of θ_1 ($n_\uparrow - n_\downarrow$).
  2. Depends strongly θ_2.
  3. Scales with s-wave scattering length $a_{\uparrow\downarrow}$.
  4. Increases with inhomogeneity as $\Delta\theta^2$.
- No trap-state changing collisions & resolved sidebands → observed predicted spin waves
- Applicable to fermion lattice clocks
  - Can often have smaller $\Delta\theta$.
- Correlations shift $\Delta\nu$=0 to θ_2=0.51π
  - Max Ramsey fringe contrast biases to colder atoms, giving cos(θ_2)=0 at θ_2=0.56π.
  - g correlated with θ_2 → lower θ_2
  - g anti-correlated $\Delta\theta$ → higher θ_2

PHARAO (10^{-16})	Shift	Uncertainty
Quadratic Zeeman	440	0.4
Blackbody radiation	-170	0.5
Ultracold collisions	-25	1.2
Cavity Phase	0.3	1
Microwave Lensing	1.2	0.3
Total		**1.7**





$$\Delta\nu = \sum_{pairs} \frac{g}{2\pi} \frac{\sin(2\Delta\theta_1)\sin(\Delta\theta_2)\cos(\bar{\theta}_2)}{\sin(\bar{\theta}_1)\sin(\bar{\theta}_2)}$$

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