stellar interferometry : an overview about basics

Yves Rabbia
Observatoire de la Côte d'Azur
UMR Lagrange, Nice, France

rabbia@oca.eu

Perso 06 24 33 84 96

stellar interferometry :
an overview about basics

sections

- introduction : a problem raised
- science context and motivation
- few academic reminders
- basics for interferometry and aperture synthesis
- limitations and subsequent needs
- interferometers : principle, production, typology
- difficulties in real world (and some remedies)
- managing with data and some results
- quick-look at some alternative HAR methods
- nulling interferometry and coronagraphy

introduction

purpose of the talk

to recall and to illustrate (hand waving as far as possible)

- general framework and some land marks
- specific terminology (and debunking "jargon")
- basics of interferometry and aperture synthesis
- few things about nulling techniques

quick look

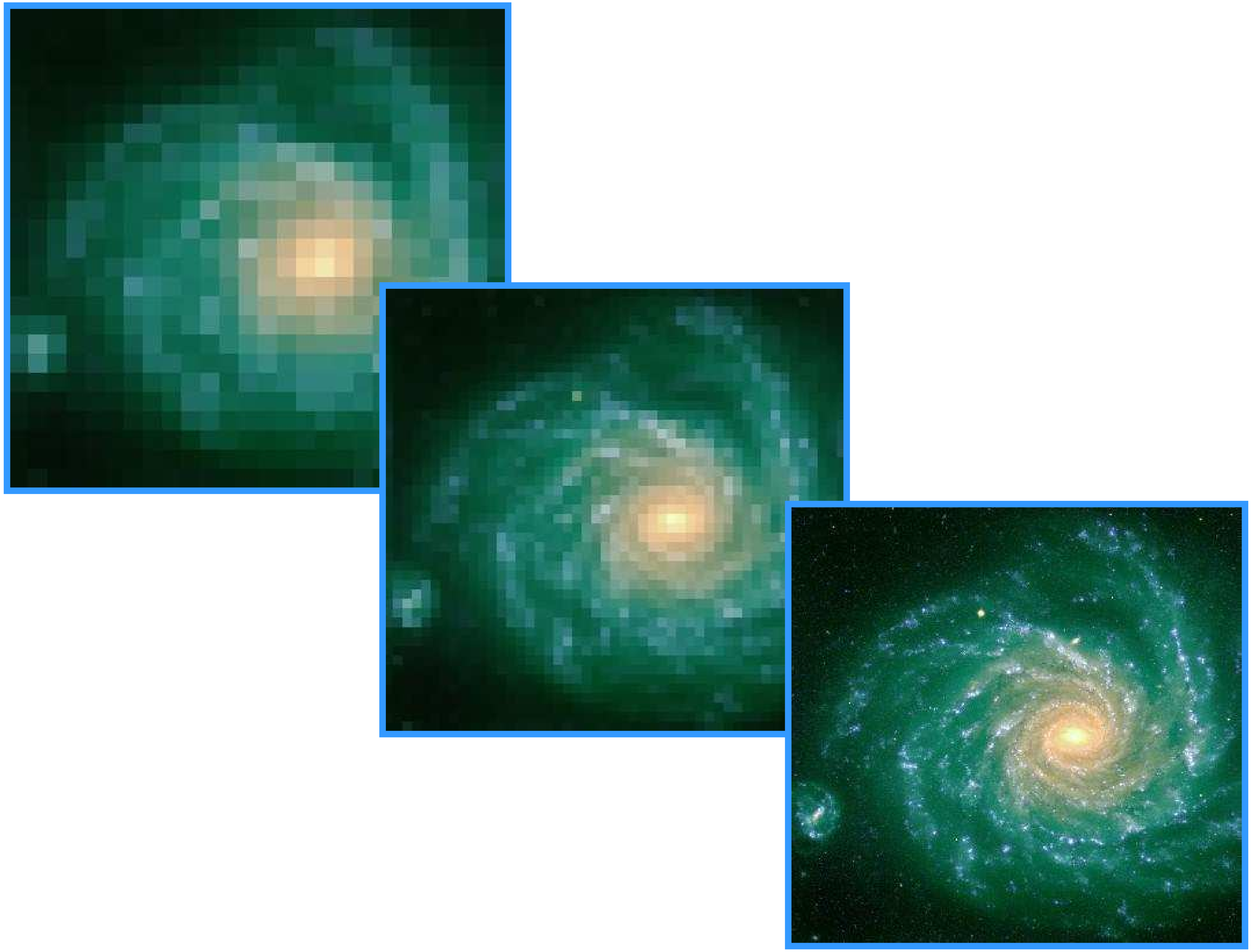
stellar interferometry is part of a large body of topics covering methods and techniques aiming at

High Angular Resolution (HAR)

the underlying goal is Aperture Synthesis
but other techniques contribute to HAR

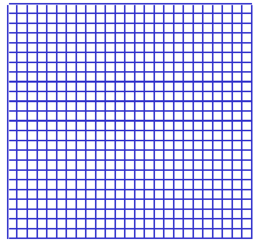
- eclipsing binaries
- lunar occultation
- speckle interferometry

interest of angular resolution : self-speaking illustration :

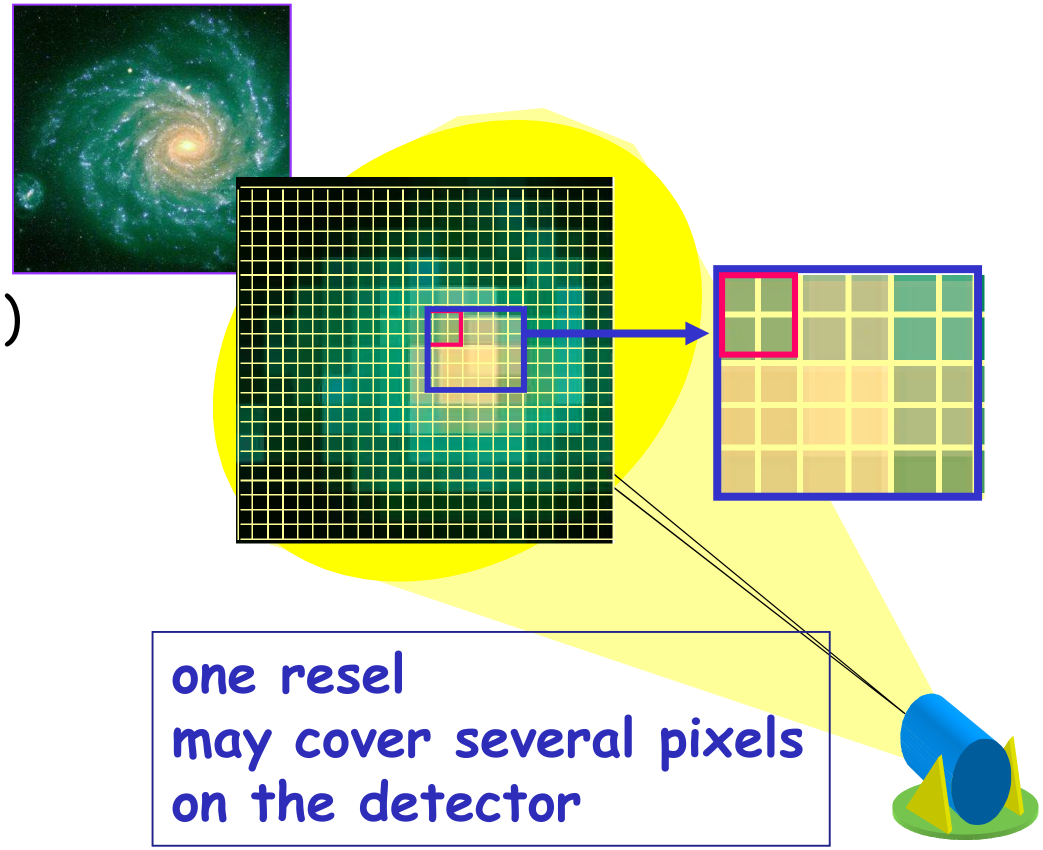


resolution and resolution

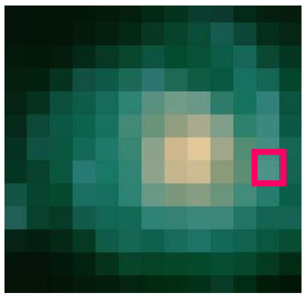
detector resolution : the number of pixels IS NOT the point



what counts for the astronomer is
the size of pixels over the sky



and this depends
on the instrument
(including observing conditions)



pixel on the sky
jargon : resel,
resolution element

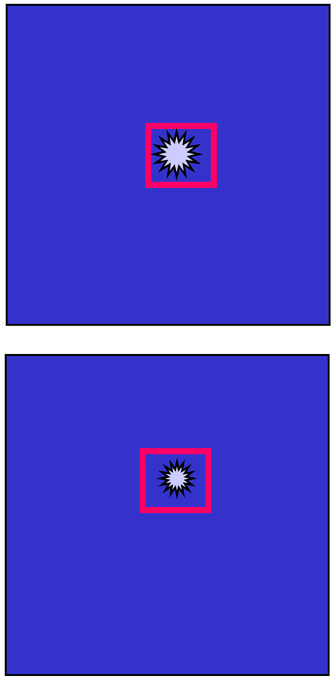
**one resel
may cover several pixels
on the detector**

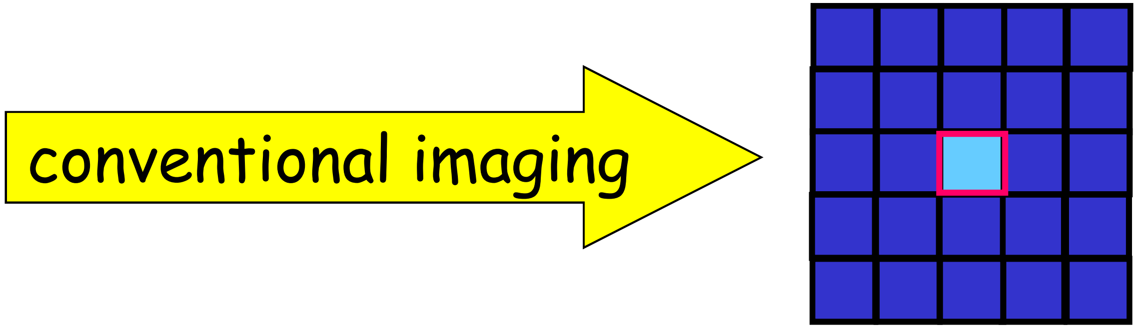
a specific example ; the case of stars

even the most reduced ambition is not achievable

conventional imaging is powerless

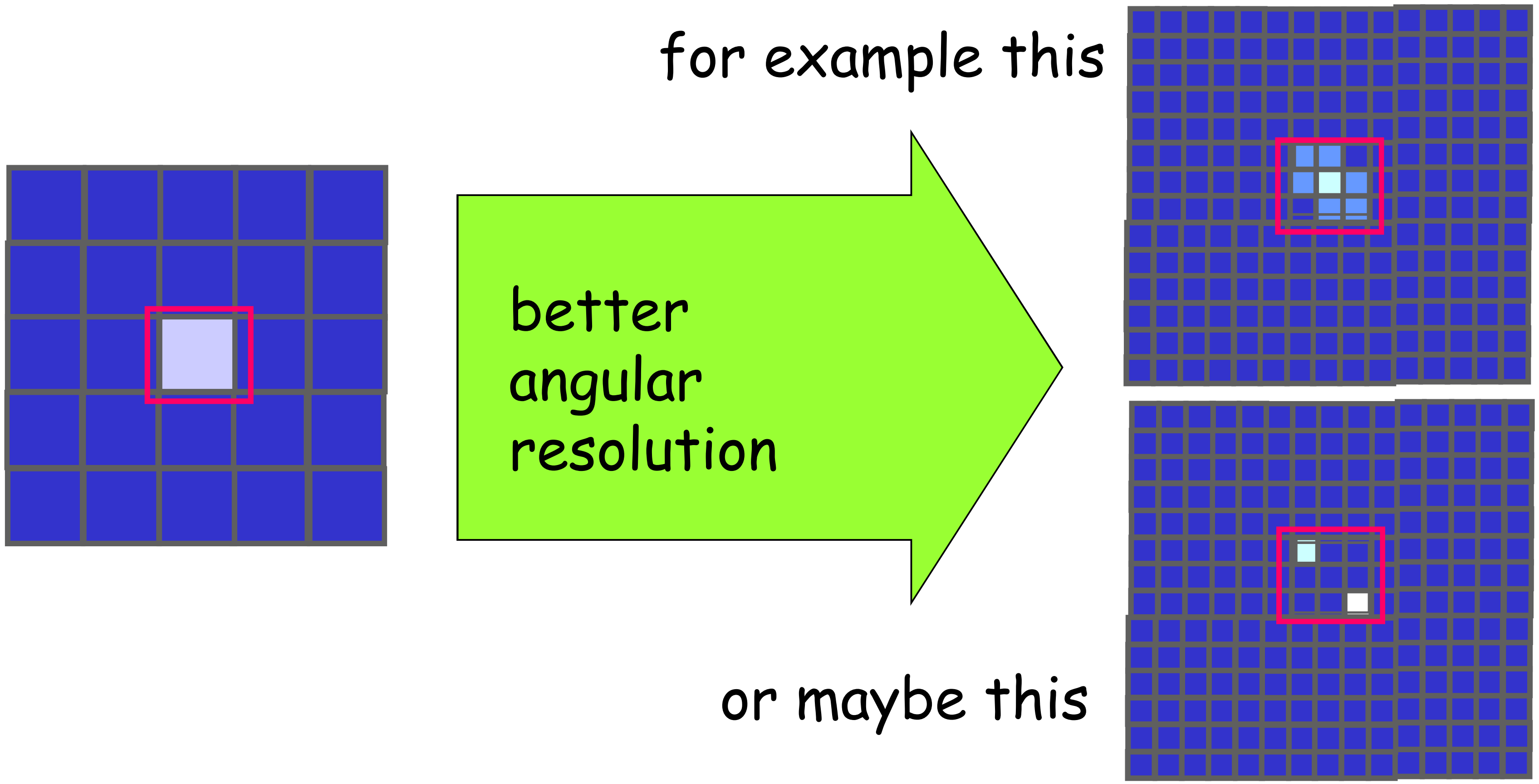
be the star 'big' or 'small' and ignoring atmospheric effects the size of an image would be reduced to a single resel





this single resel situation is caused mainly by the extremely small size of the target but also by unsufficient angular resolution

an exemple of improved angular resolution the morphology of the source begins to appear



solving the problem ??

conventional imaging being disqualified
it is necessary to find another way to describe
the brightness distribution of the star
(or at least to determine some of its key parameters)

a relevant method is named : **aperture synthesis**
it is based on **interferometry**
which is one of the techniques pertaining to
High Angular Resolution

specific words that we are going to explicit

specific terminology

interferometry

interfero → interferences

metry → measures

measuring something by using interferences

interference : mutual influence of two things

interference : mostly pertaining to wave phenomena

here we shall consider interferences of light
for which we use the model of electromagnetic waves

a definition (?)

here we speak of stellar interferometry which means

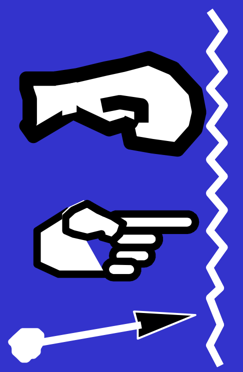
" measuring something of a star by using
interference of the light
received from this star "

stellar interferometry :
an observational method in astrophysics,

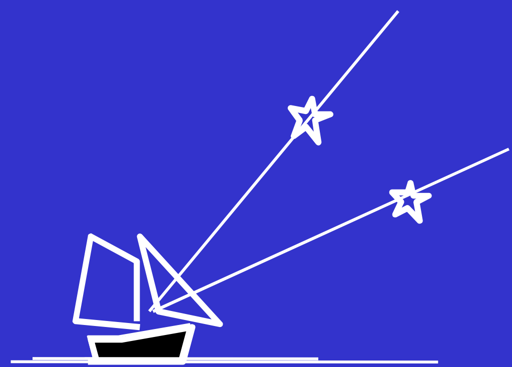
based on interferences and **coherence** of light

which aims at obtaining **high angular resolution**

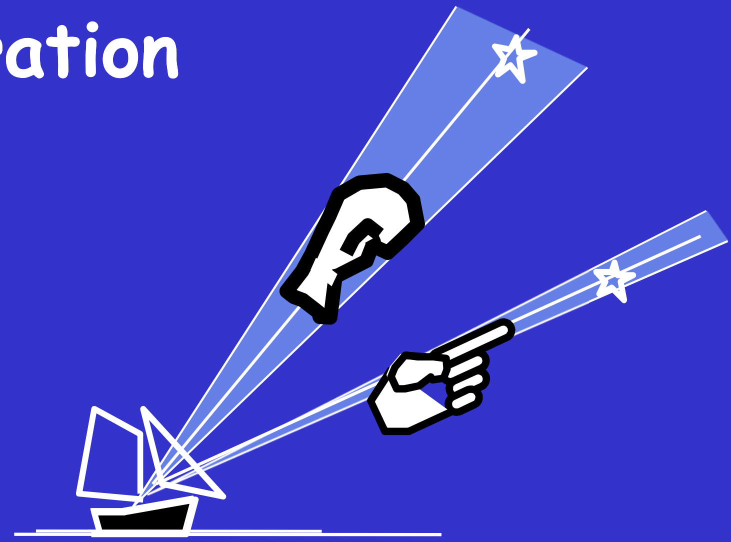
High Angular Resolution : a matter of "finesse" in exploration



resolution



angular



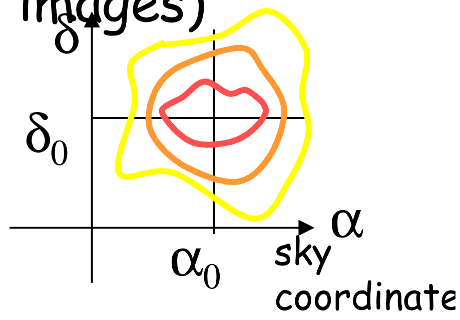
"finesse" depends on the instrument used

what does "high" mean ?

it means "responding to current scientific needs"

the purpose of angular resolution in astrophysics

describe the **brightness angular distribution** of a source
and obtain morphological parameters (not necessarily images)

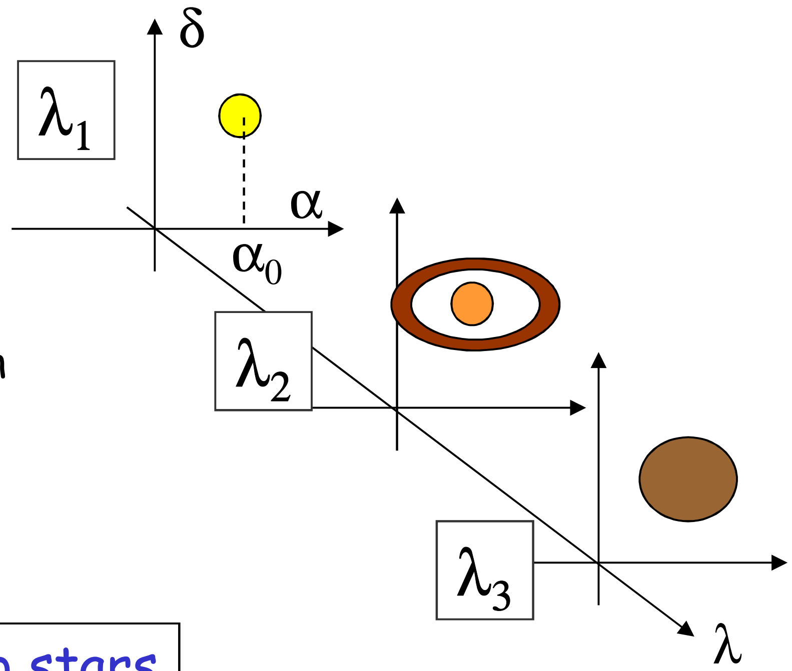


Astrophysics wants

$$I(\alpha_0, W, \alpha-\alpha_0, \lambda, t, \vec{P})$$

interferometry focuses primarily on
brighness distribution

$$I(\alpha_0, W, \alpha-\alpha_0, \lambda, t, \vec{P})$$



then on
chromatic dependance of the brightness distribution

then eventually, look for time evolution and polarisation

in the following , we restrict the topic to stars

testing your own angular resolution

not much

yet poor

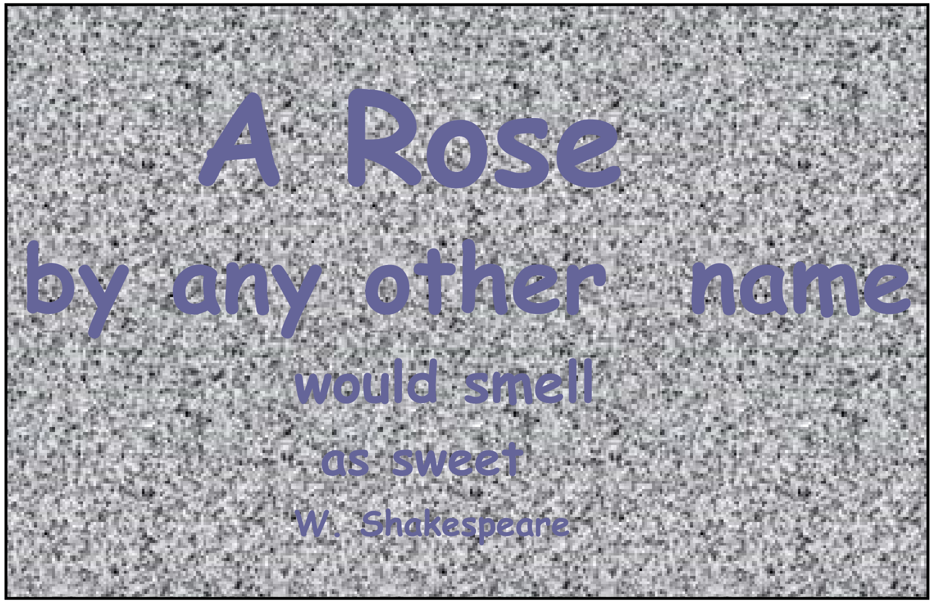
not bad

pretty much

really impressive

wooah ! fantastic

better and better



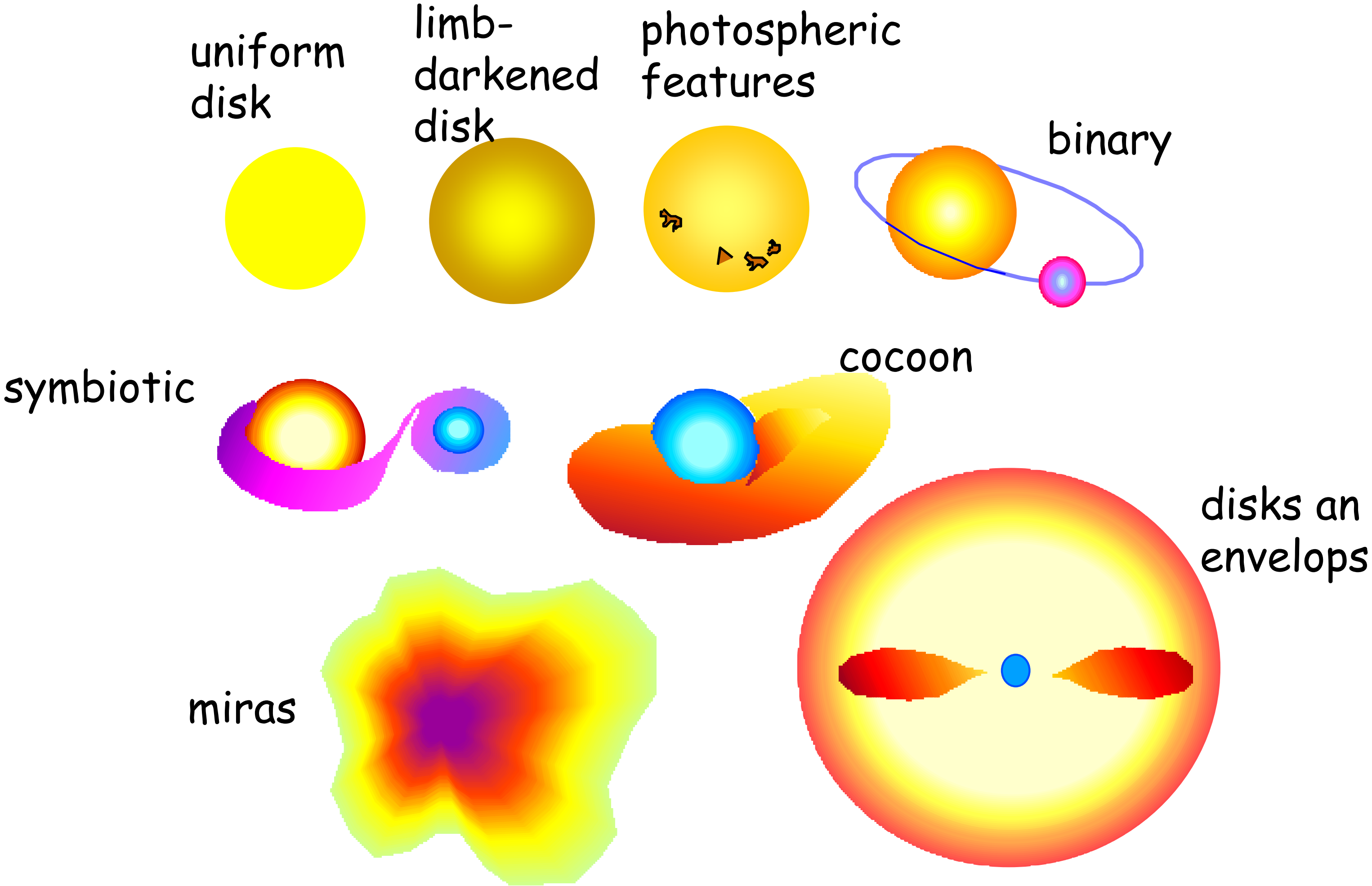
warning :

sensitivity and contrast does matter

(Signal to Noise Ratio)

science context and motivation

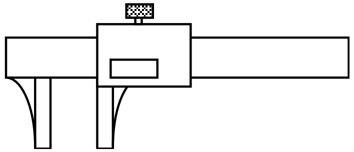
morphology ??? few examples for stars



a gradation of needs

depending on sources, few parameters might be of great help
but some morphologies would requires images

examples



binaries : angular separation (vector)

single stars : angular diameter

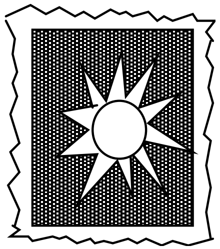
limb darkening : radial variation of intensity

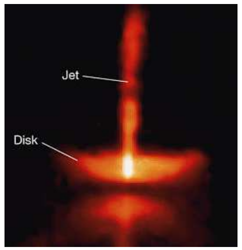
multiplicity : number of components, geometry

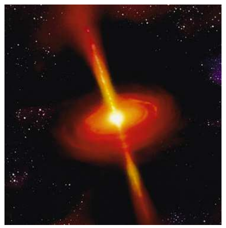
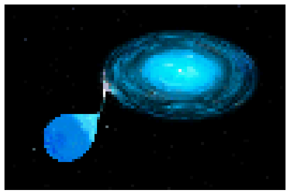
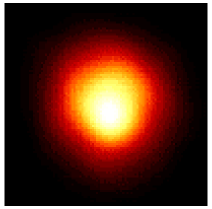
extended atmosphere : radial structure, schock wave

circumstellar matter : angular diameter, structure

complex objects (Miras, bipolar jets, disks, ...)





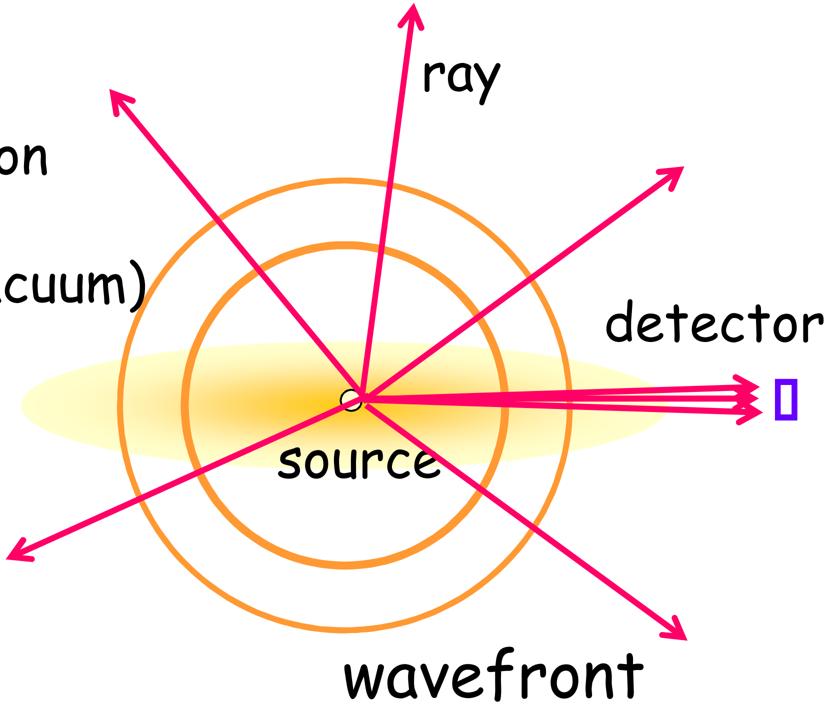
few academic and elementary reminders pertaining to our topic

light
phenomenology of interferences
optical index
diffraction

light : a tentative definition, initial intuitions and synthesis

light :

propagation of energy or information
between two points in a medium
(the latter could possibly be the vacuum)



propagation how ?

intuition 1
rays of light
rectilinear
propagation

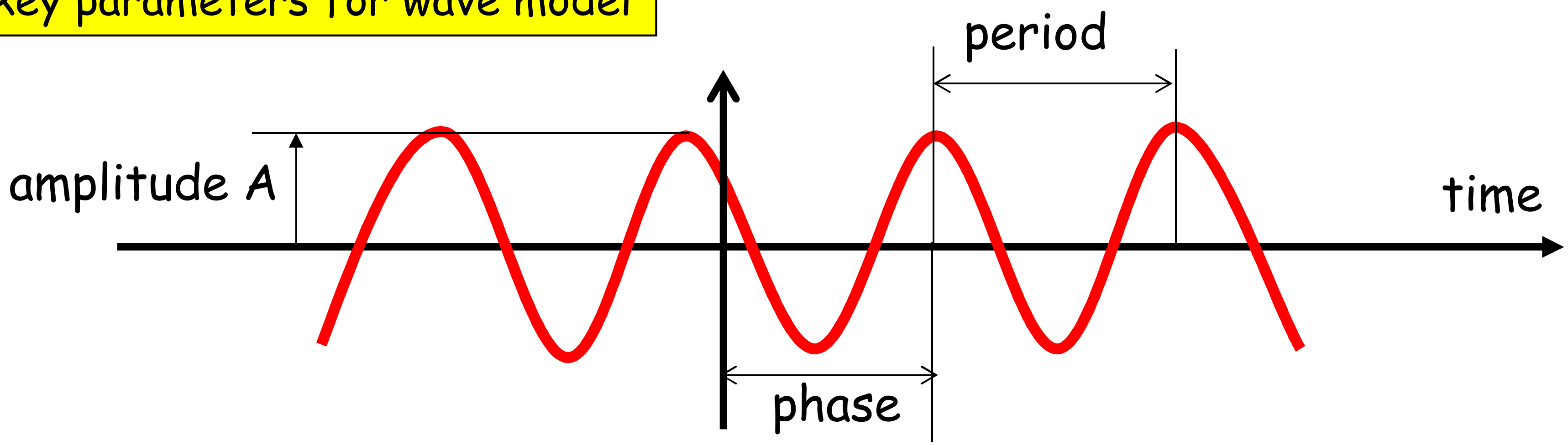
intuition 2
energy front
expansion in
space

Fig. 2. — Ondes transversales à la surface de l'eau.

the rays illustrate
the direction of propagation of
the energy conveyed by the
front

rays are locally perpendicular
to the wavefront

key parameters for wave model



frequency ν : number of periods per second
(number of oscillations)
unité Hertz ou s^{-1} (visible light: $3.\ 10^{14}$ Hz)

amplitude A : a parameter related to the collectable energy :

E proportional to (A^2)x(recording time) unité : joule

also is introduced **the wavelength** λ :
distance travelled by the energy during one period

and **the phase** : angular parameter linked to an origin of time

key parameters for LIGHT

propagation velocity : in vacuum c = 3.10^8 m/s,
in a medium of optical index "n" : v = c/n

frequency ν:

NOT depending on the medium where propagation takes place

with the « photon model », frequency appears in the energy of an individual photon:

E = h.ν Joule where h = Planck's constant 6.62*10^(-34) Joule.seconde

wavelength λ

$$\lambda = \frac{propagation\ \ velocity}{frequency}$$

depending on the medium where propagation takes place
via the optical index « n » (refraction index) of the medium

in a medium of
optical index « n »

$$\lambda_{medium} = \frac{velocity\ \ \mathrm{v}}{frequency\ \ \nu} = \frac{c}{n}.\frac{1}{\nu} = \frac{\lambda_{vacuum}}{index\ \ n}$$

in vacuum , v = c; n=1

$$\lambda vacuum = \frac{velocity\ \ c}{frequency\ \ \nu}$$

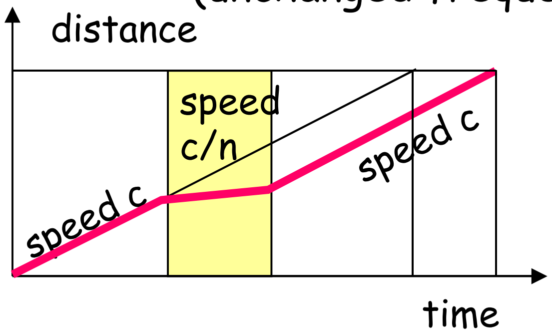
illustration : influence of the optical index

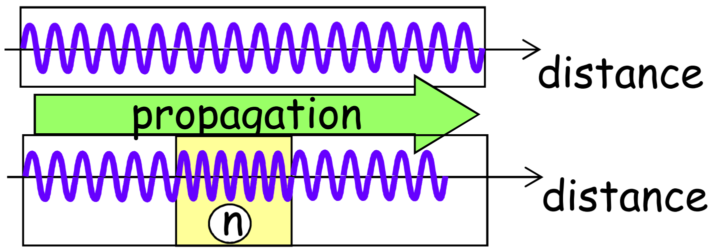
propagation velocity is reduced because of « n »

$\lambda_{milieu} = \lambda_{vide}/n$
and $v = c/n$

geometrical path L
optical path n.L

in the medium light walks at the same pace as in vacuum (unchanged frequency) but with shorter steps

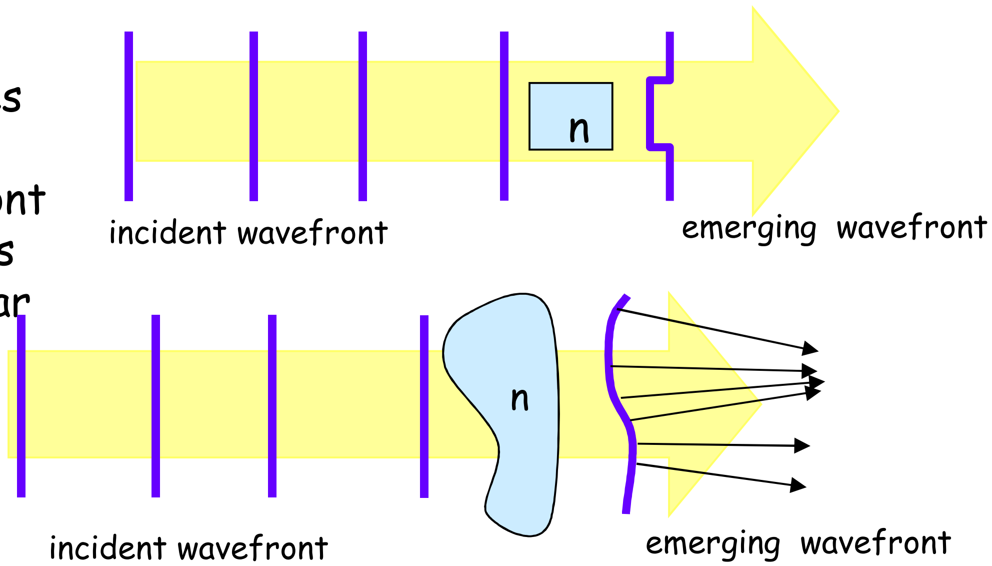




to travel a given distance
it takes more time in a material medium

wavefront distorsions

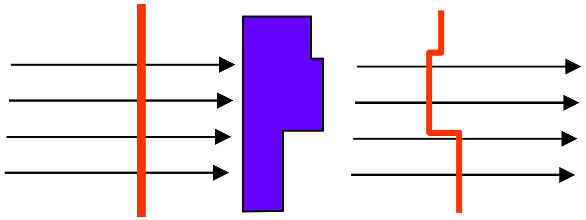
An index inhomogeneeeity locally modifies the propagation velocity what is inducing geometrical distorsions over the wavefront the distorsions induce optical aberrations since the rays remain locally perpendicular to the distorted emerging wavefront

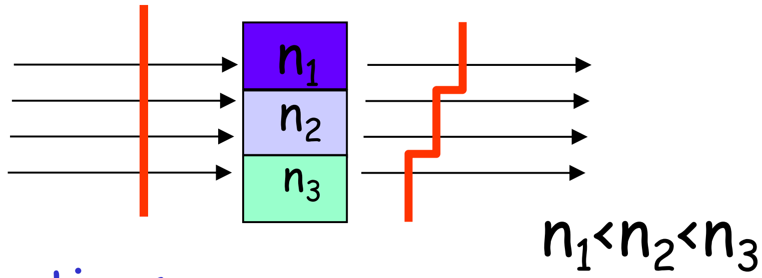


Malus Dupin theorem, 1808,
a conseqsuence of the Fermat principle

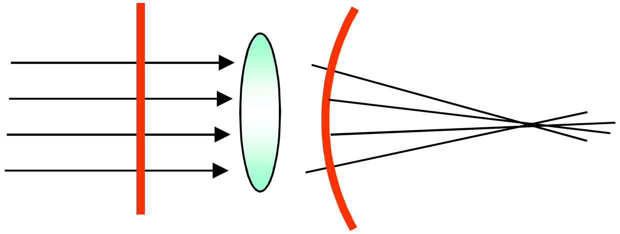
illustration : distorted wavefront

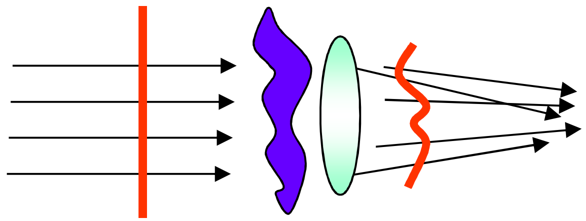
influence of the geometry and of the index of the crossed medium





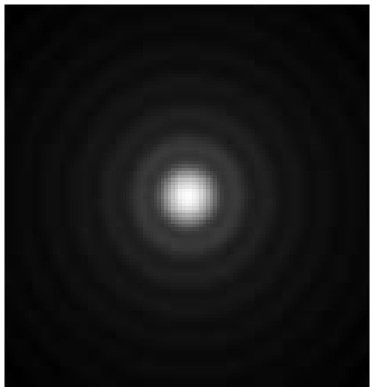
wavefronts with various corrugations

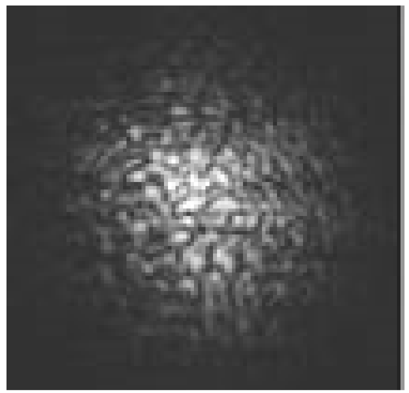




no distorsion : ideal image

distorted wavefront : aberrated image



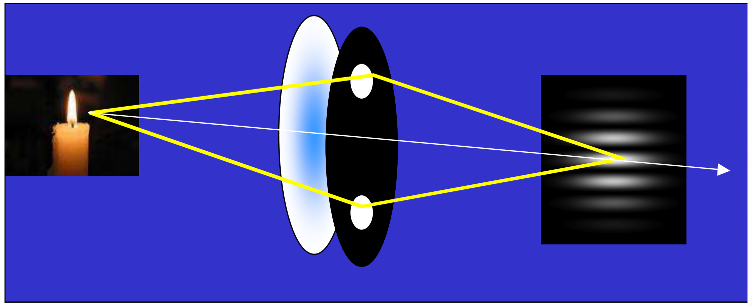


phenomenology of interferences and behaviour of waves

an historical device : Young's fringes

a set-up for legend

fringes :
alternated
bright and dark areas



thomas young
1773_1829

dark !? paradoxical ?

??

interferences :

light + light → light
light + light → darkness
energy + energy → energy
energy + energy → NO energy

paradox ? the key for darkness : unmatched optical paths

more light

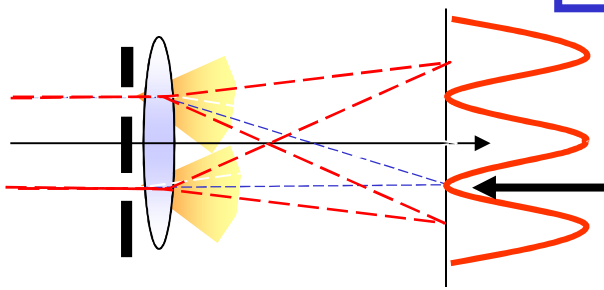
light + light

no light

more energy

wave 1 + wave 2

no energy



path difference zero

path difference λ / 2

path differencer λ

wave 1, **path length L1**

wave 2, **path length L2**

energy

no energy

waves in phase

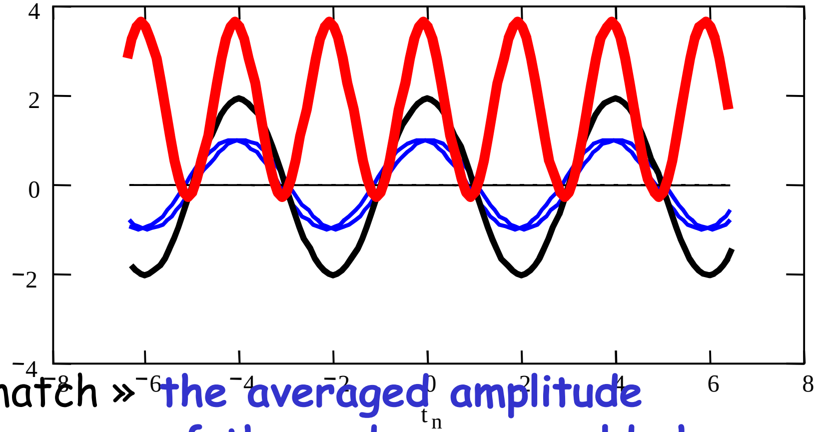
$L1 - L2 = n.\lambda$

$L1 - L2 = n.\lambda + \lambda / 2$

waves in opposition of phase

addition of waves and observed energy distribution

addition of two sinewaves « in phase »
(no pathlengths difference)

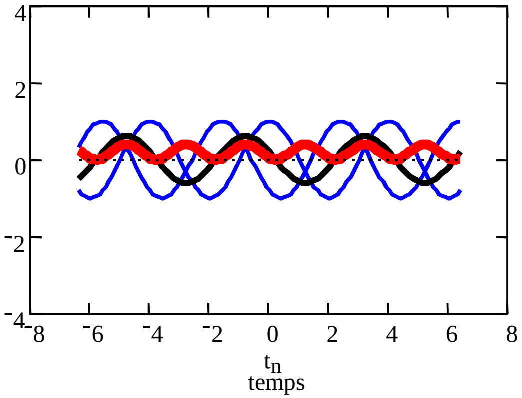


amplitude of the sum

energy distribution of the sum
(after averaging square modulus)

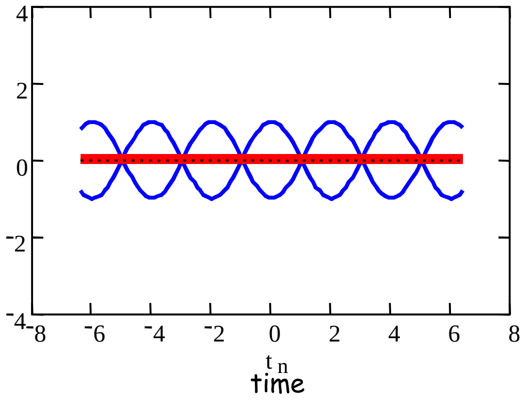
addition of two sinewaves with a phase mismatch »
(non –zero pathlengths difference)
less collectable energy

the averaged amplitude of the red curve would show the level of collectable energy when adding two waves



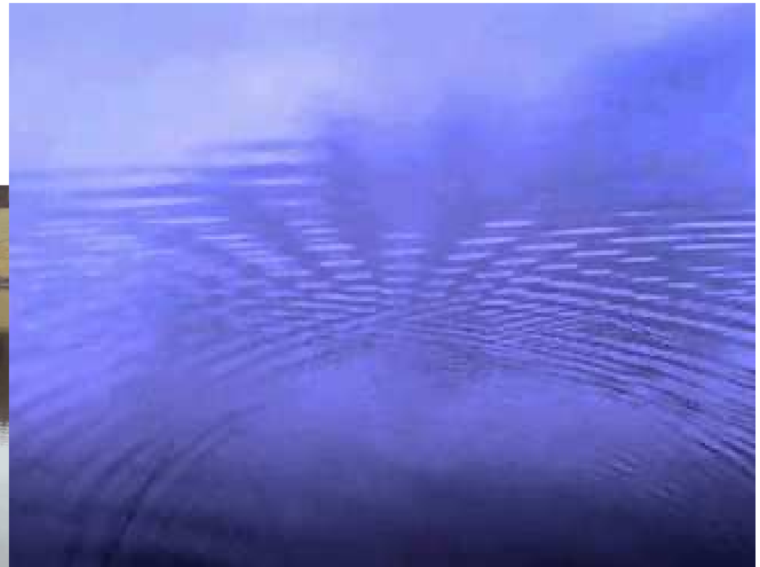
addition of two sinewaves
« in opposition of phase » :
no energy collectable » :

addition of the waves

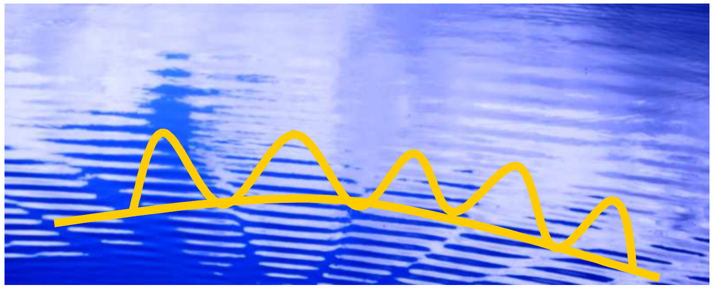


fringes « live » 01 (aquatic experiment)

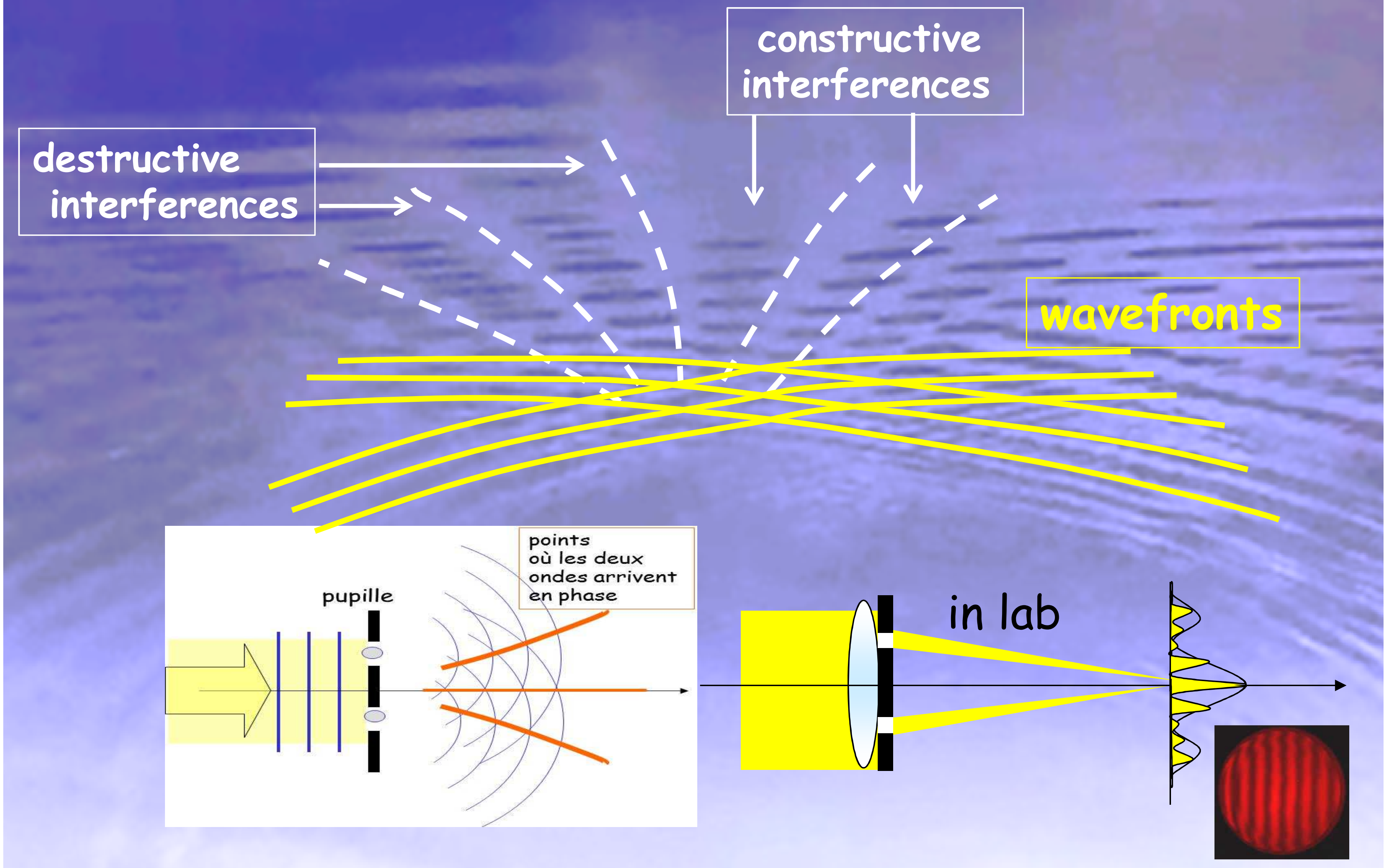
little quiet pond
and little stones

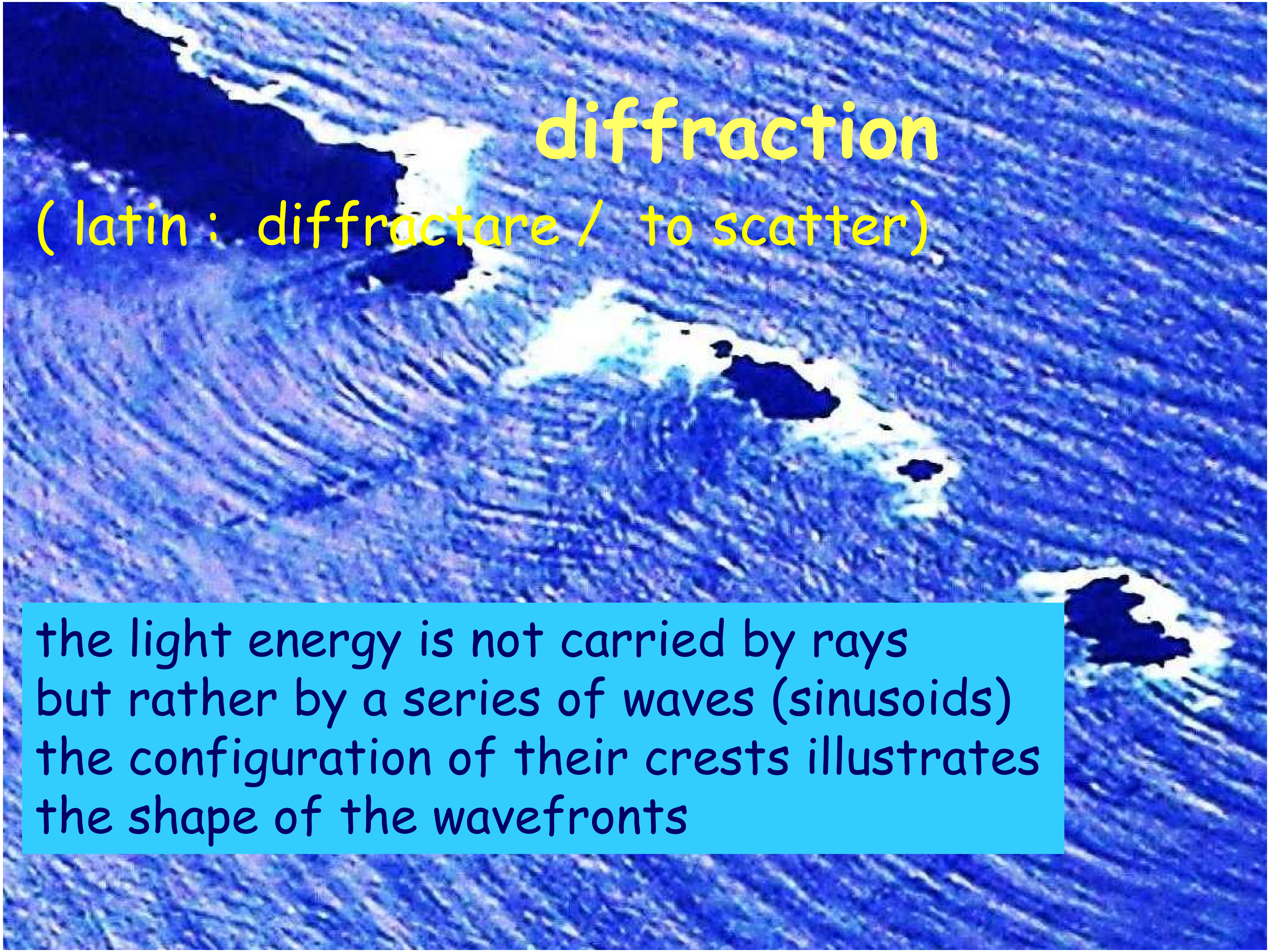


recorded fringes « live » 02



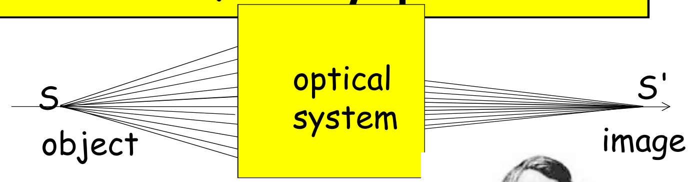
fringes« live » 03 : interpretation



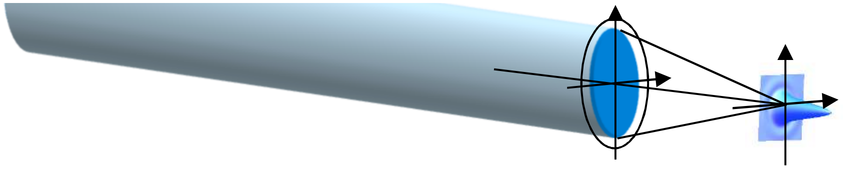

diffraction
(latin : diffractare / to scatter)
the light energy is not carried by rays
but rather by a series of waves (sinusoids)
the configuration of their crests illustrates
the shape of the wavefronts

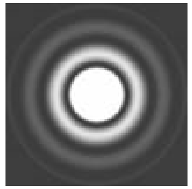
ideal image of a point source : diffraction, Airy pattern

in geometrical optics the image of a point is a point



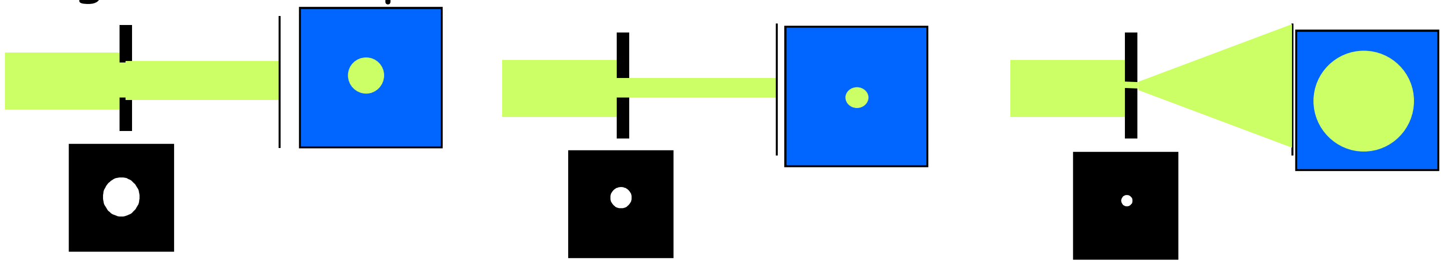
the observation shows that this is not the cas.
The image is a patch named « Airy pattern »

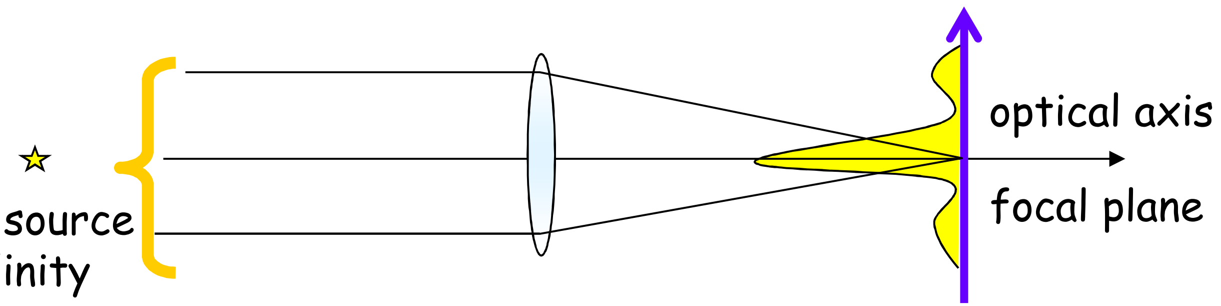


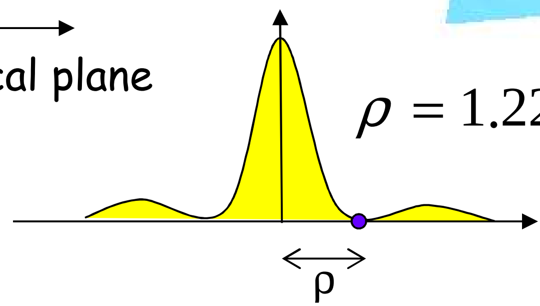


George Airy
(1801-1892)

this situation results from the wave nature of light and the phenomenum called « diffraction »







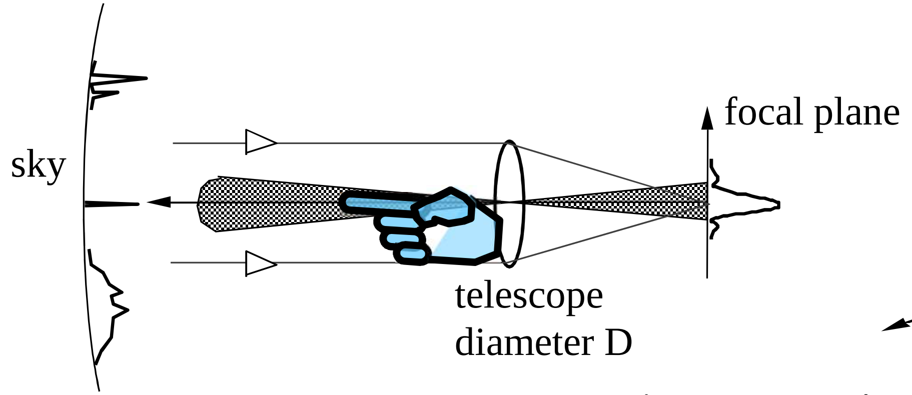
$$\rho = 1.22 \frac{\lambda}{D}$$

« finesse » in image analysis, exploration lobe

exploration lobe : report back to the sky of the angular distribution given by the instrument for a point source

also named : Point Spread Function (PSF)

note : angular extension of the lobe ≈ wavelength / diameter



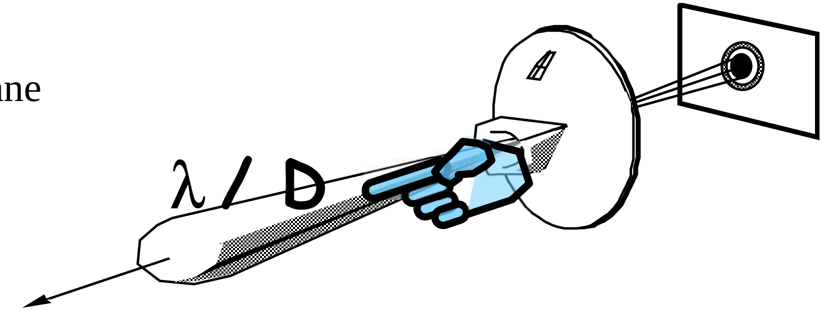
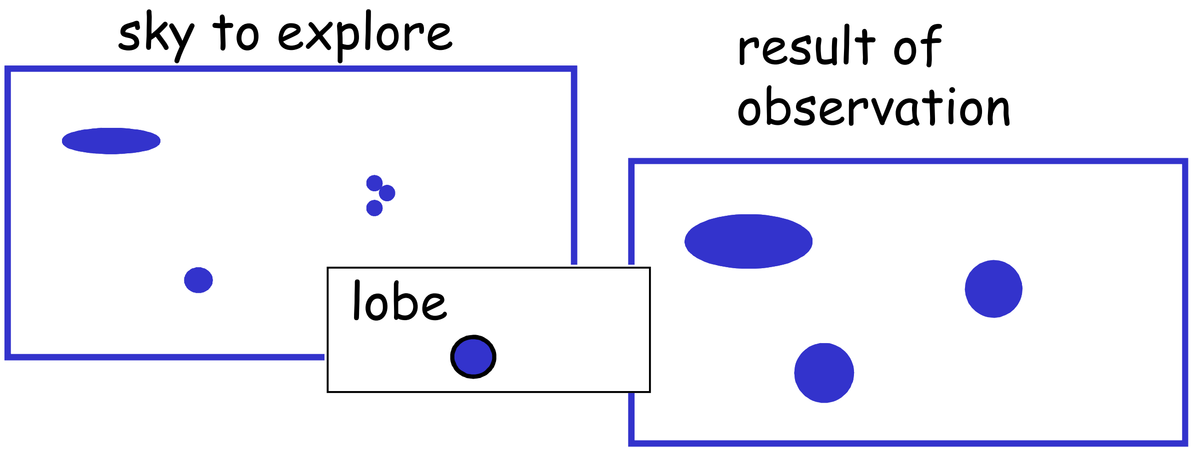
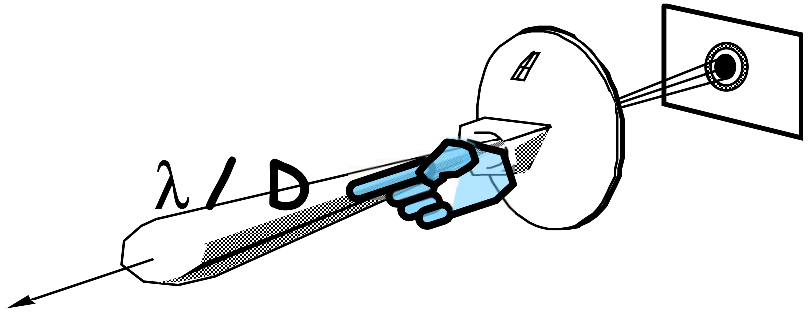


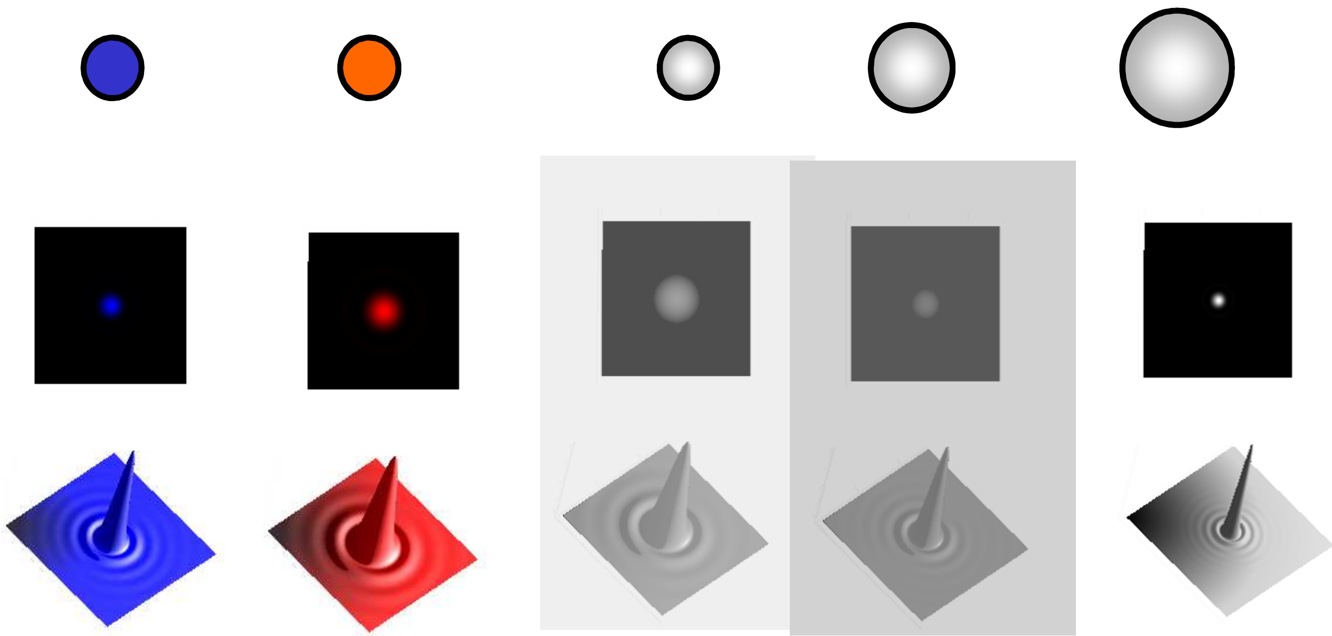
illustration
as lived



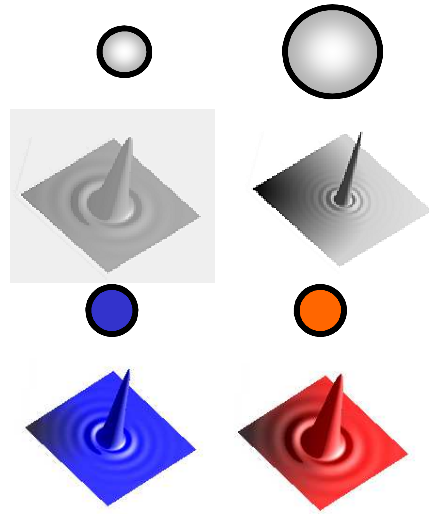
pictorial illustration : theoretical behaviour of the exploration lobe

$$\alpha \propto \frac{\lambda}{D}$$





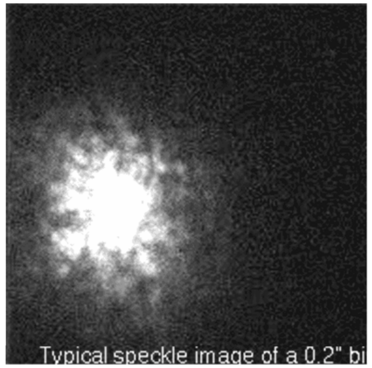
immediate limitations



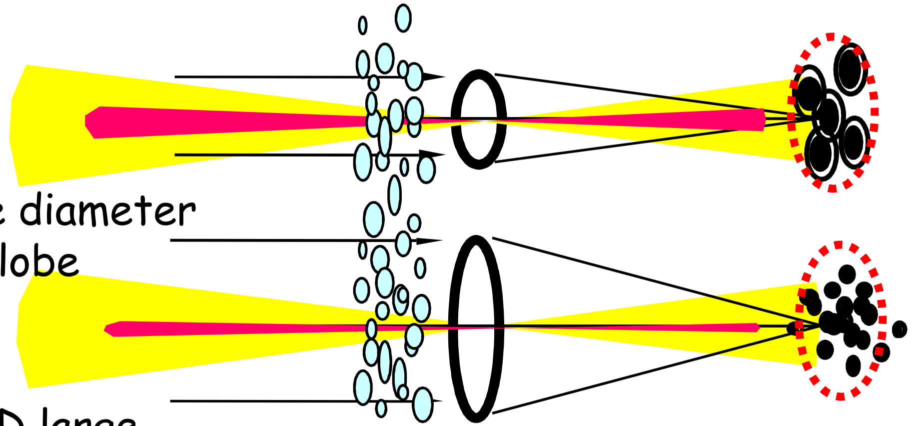
instrumentation

lobe extension λ/D and looking for 1 milliarcsec
lead to D = 100 m in the visible (and more in Infrared)
telescope not (yet) available

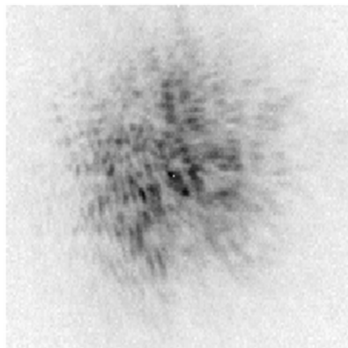
observing conditions



mainly atmospheric turbulence
PSF degraded , loss of resolution (speckles)



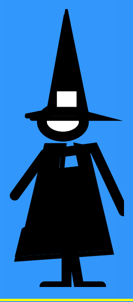
theoretically
the larger the diameter
the finer the lobe



but D small or D large ,
extension of PSF quite the same

problem for imaging stars !!

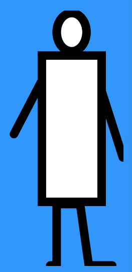
angular dimensions of stars are so terribly small



astronomers (scientific requirements)

need High Angular Resolution

I want milliarcsec level $5\ 10^{-9}$ radian



engineers (technical requirements)

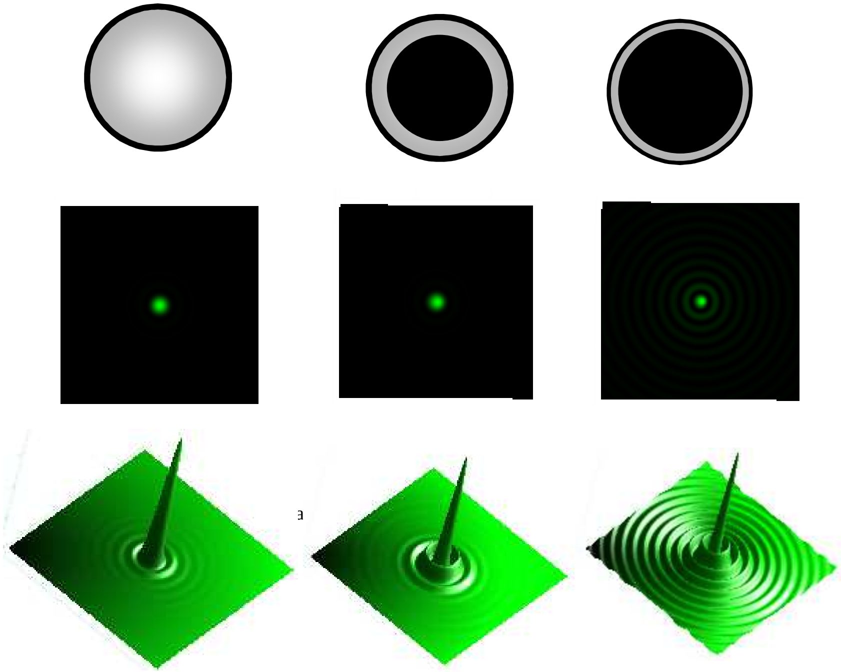
hep ! : diffraction sets a limit λ / D

what leads to several tens of meters for D

interferometry aims at breaking the limits of conventional imaging
so as to determine some morphological parameters
without large telescope diameters, (and ultimately produce images)

phenomenology_ 1
on the way towards interferometry (pictorial)

actually it is the peripheral corona of the collector which governs the angular extension of the central peak of the Airy pattern
yet at the price of increased sidelobes (and less photons, of course)



but ..
not easier to make
large corona
than large diameter !

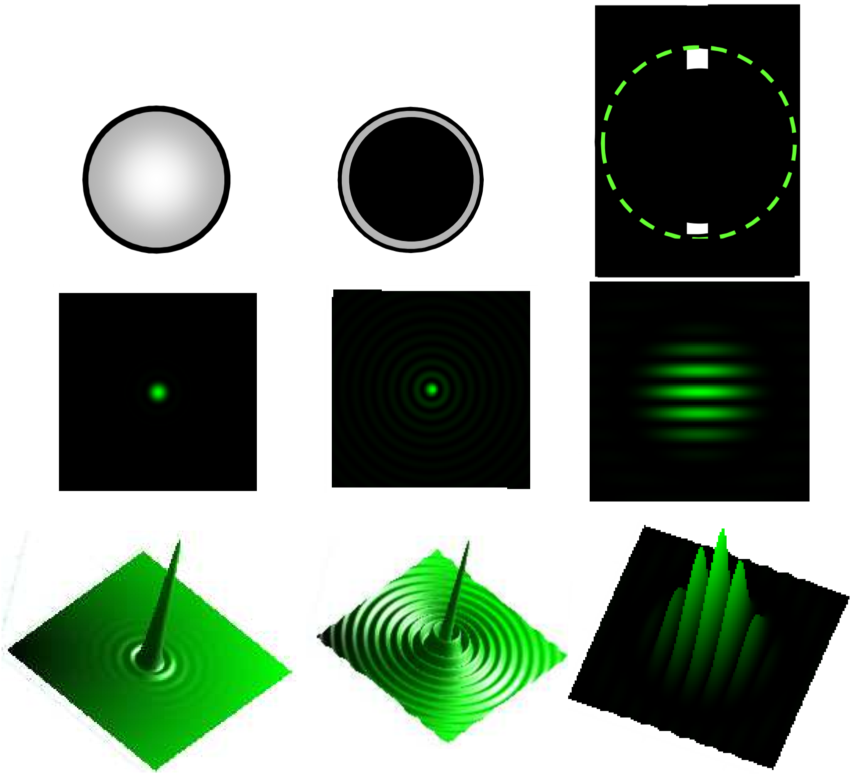
so what... ?

phenomenology_ 2

not easier to make large corona than large diameter !

but ... look

let's take two pieces of the corona



in one direction
the central peak remains narrow

however
on the perpendicular direction
the peak extension is the one
of the small piece

fringes and interfringes (spatial period)

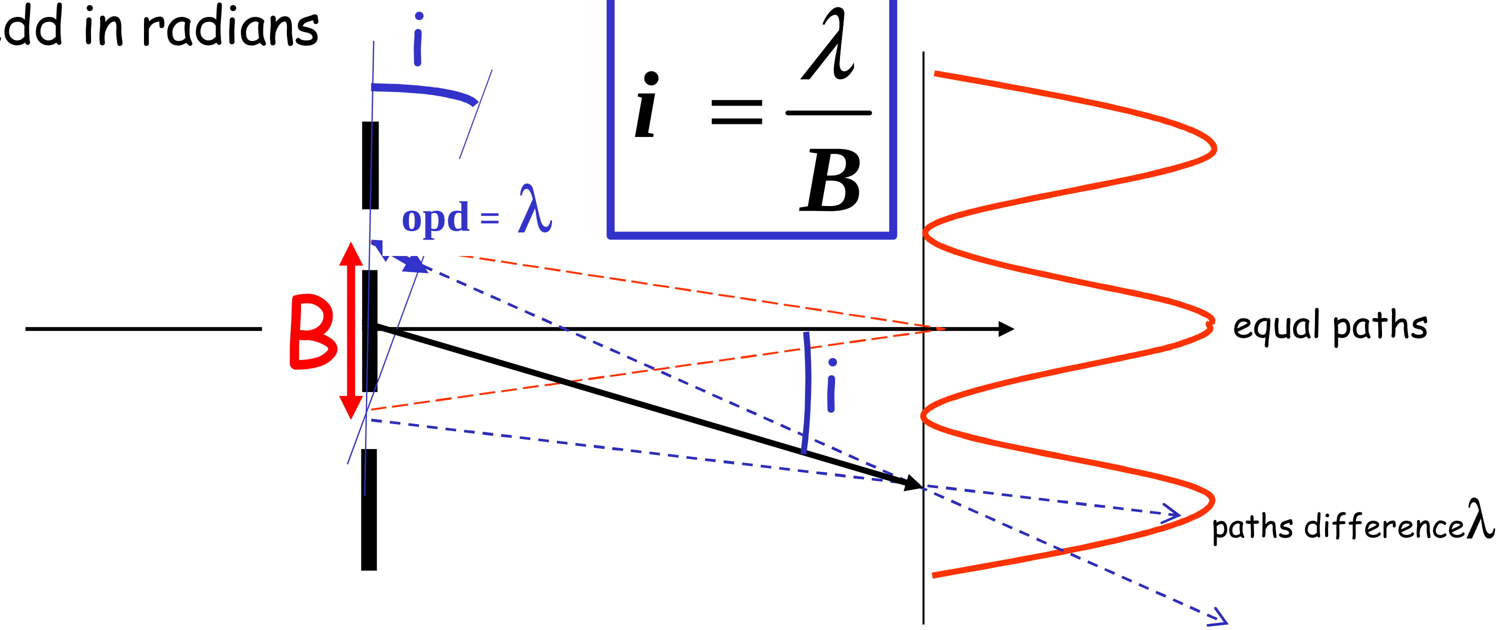
the period of fringes is governed by the « optical path differences »

bright fringe if $opd = k\lambda, \quad k \ integer$

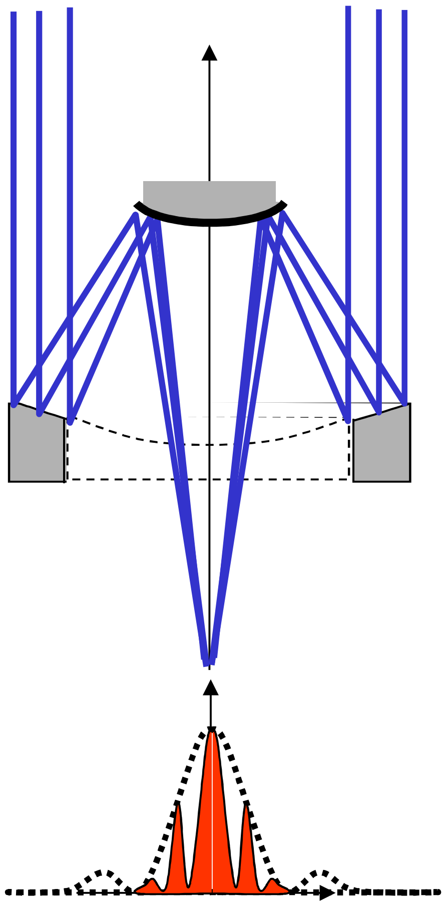
dark fringe if $opd = (2k+1).\frac{\lambda}{2}, k \ integer$

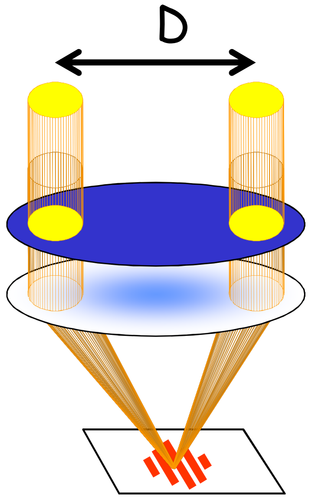
here we show the angular interfringe
it is expressedd in radians

$$i = \frac{\lambda}{B}$$

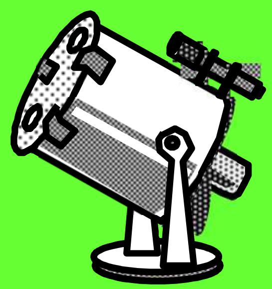


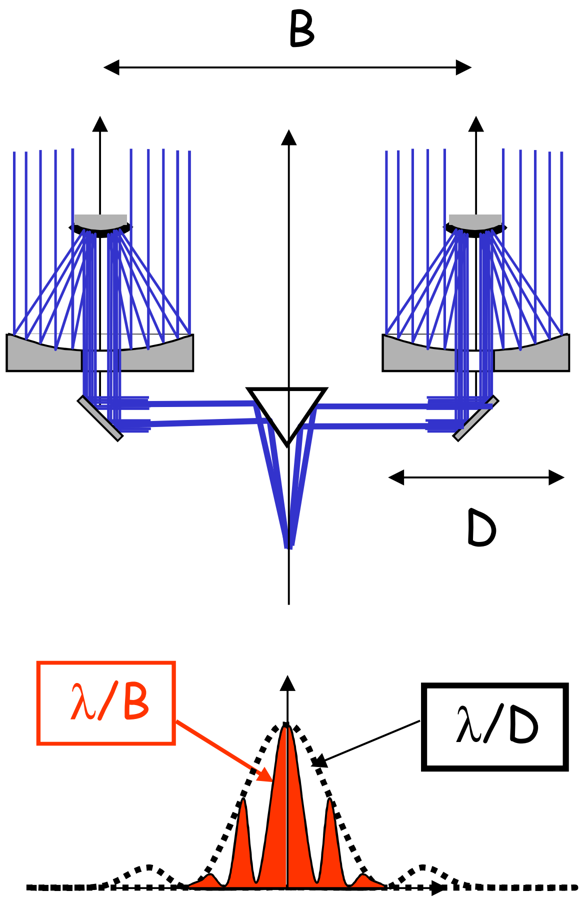
how to make it practically ?





do it yourself !





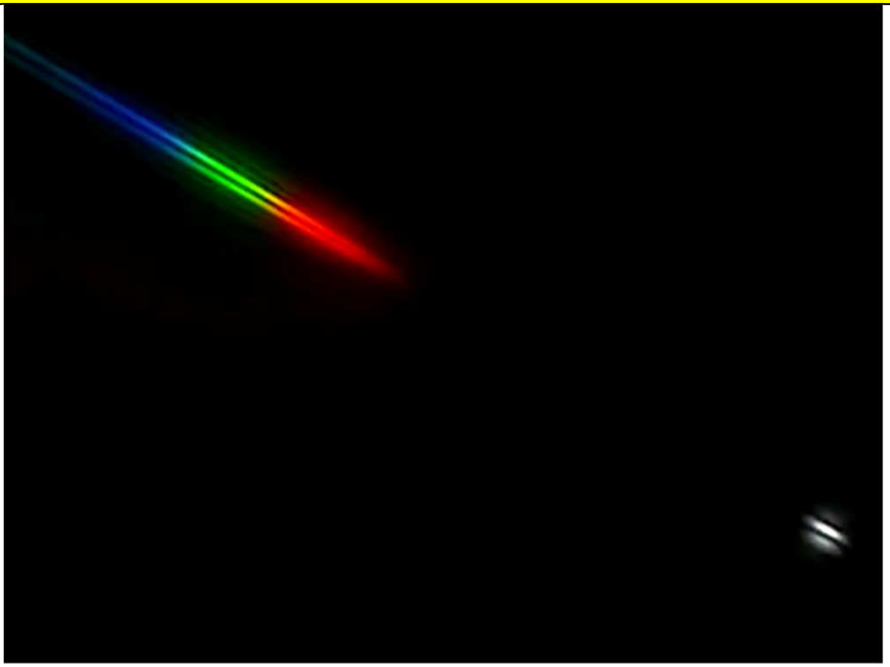
where is the gain ?

λ/B instead of λ/D

along one direction over the sky

fringes observed with a remote source

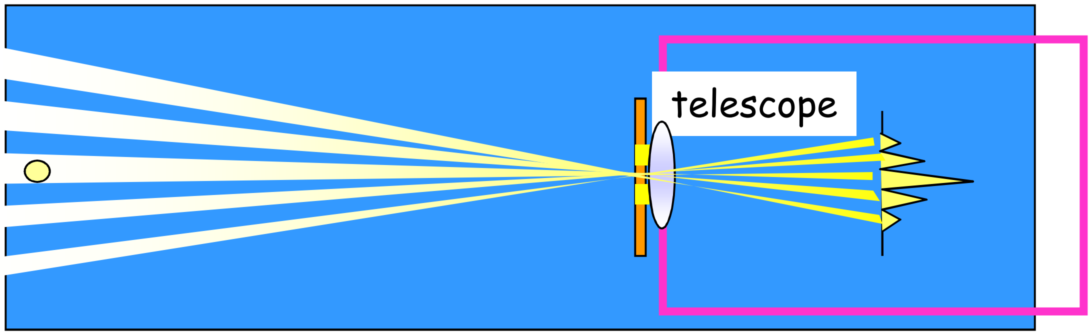
sun reflected by the roof of a car (parked in the distance)



questions appear

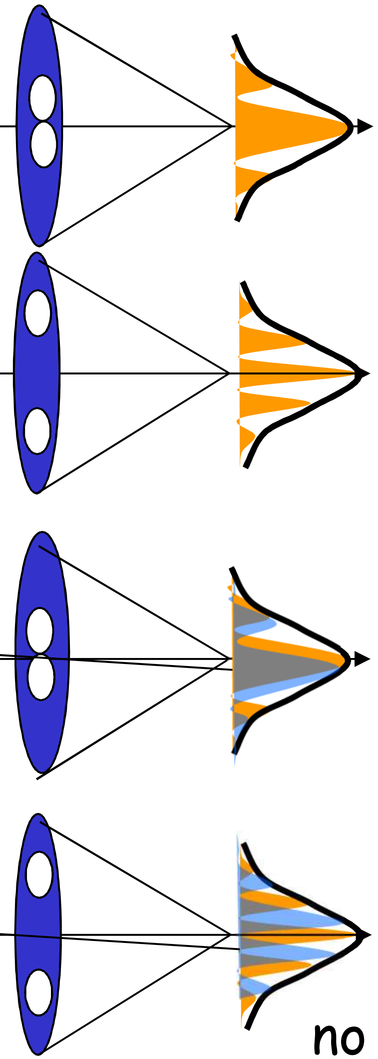
extension λ/B much finer than λ/D
gain B/D example 100m/1m
OK but, the exploration lobe is like a comb

where does it come from ?
what does that mean "exploring with a comb ?"
exploring with a lobe both narrow and large



some theoretical basis needed
but before getting these basis
let's look again a little more at phenomenology

exploring and measuring with a "fringed lobe"



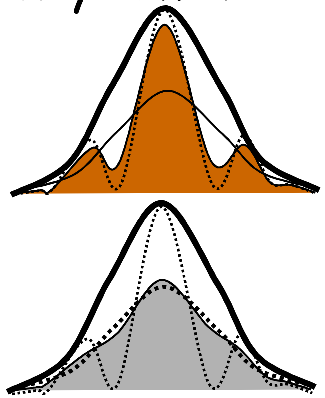
point like and small baseline

fringes with
full contrast
(max = 1, min = 0)

enlarging baseline
fringe spacing λ/B decreased

binary
fringe patterns
slightly shifted

yet fringes but
slightly lowered contrast



enlarging baseline
fringe patterns fully overlap

there angular separation
is measured : $\lambda/2B$

no more fringes, total blurring
contrast fully destroyed

basics for interferometry and aperture synthesis

toolbox and terminology

- mathematical tools
  - Fourier world (reminders)
  - spatial frequencies & spatial spectra
  - Fourier optics
  - linear filtering
- fundamentals principles
  - detection, coherence, VCZ theorem
- interferometers

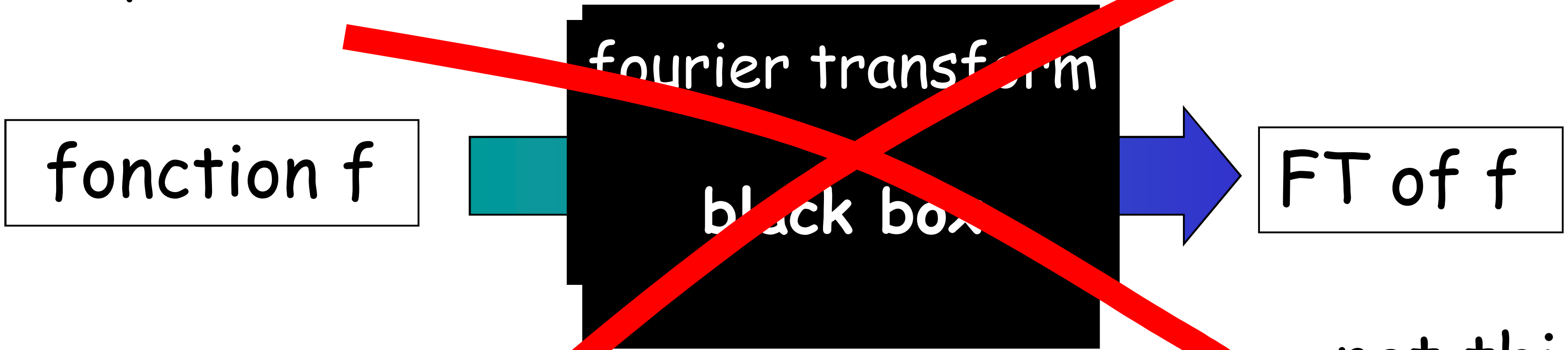
Fourier world
concept and formalism

an omnipresent tool :
signal processing, spatial frequencies,
linear filtering, coherence,
interferometry, aperture synthesis,.....

fourier, some reminders

M

a way to start ?



not this one

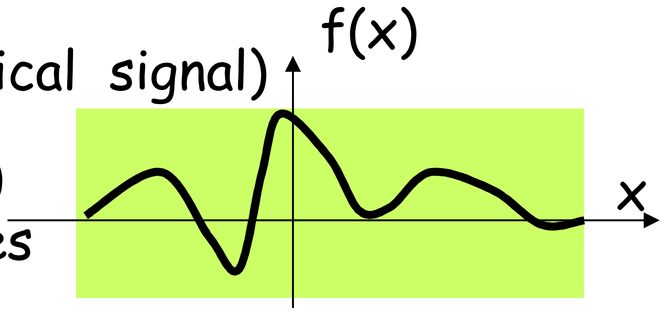
another way

$$\hat{f}(u) = \int_{-\infty}^{\infty} f(x).e^{-i.2\pi.u.x}.dx$$

not this one either
but we will come back to it later on

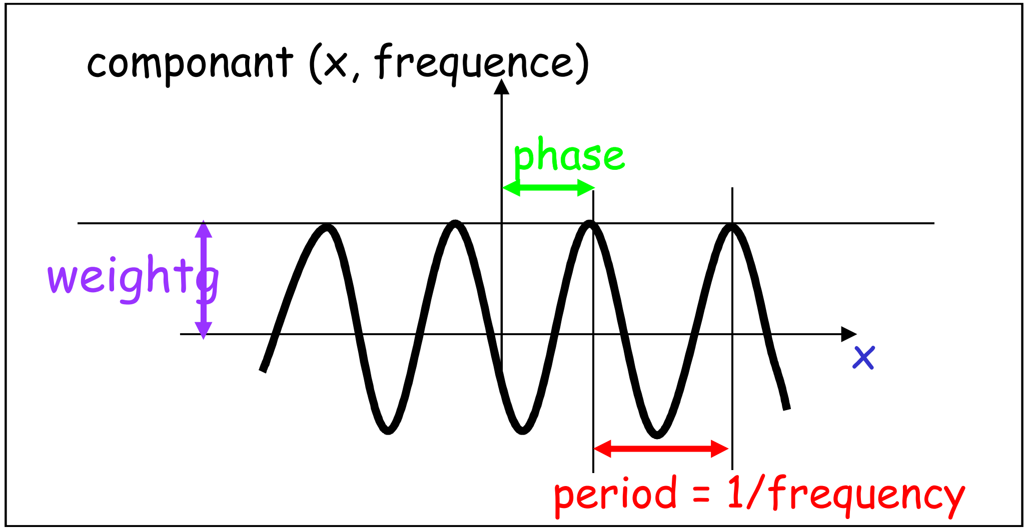
a coarse conceptual short cut

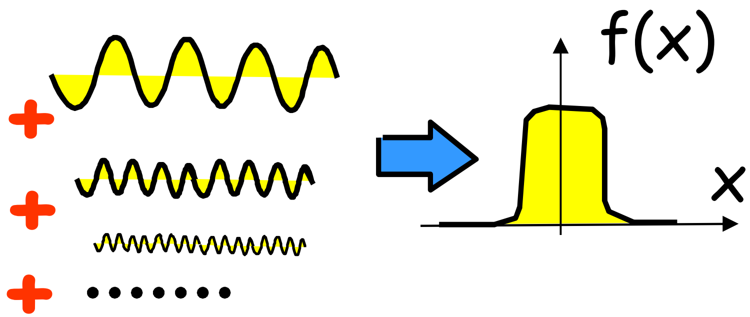
there are functions « nice" or « friendly"(physical signal)



they are described by a set of couples (x, f(x))
it is the description in the space of coordinates

there is another description that is built in the space of frequencies
description by a weighted sum of sine functions of various frequencies and phases (they form a «mathematical base » nearly like for vectors

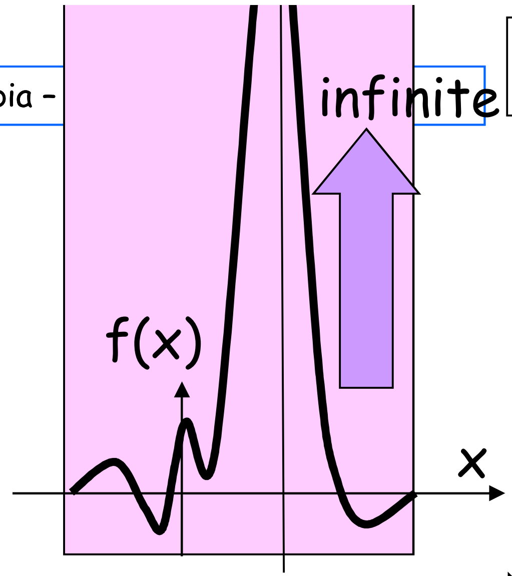


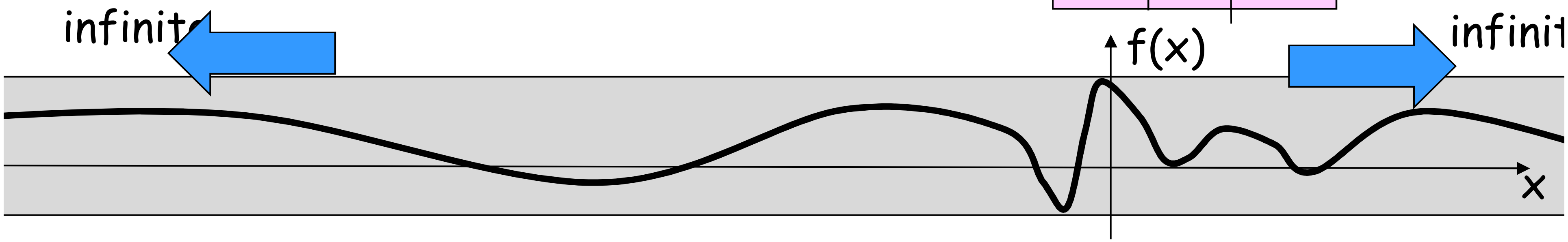


supplementary note _1

who are the « not nice» or « nasty » functions ?

(i.o.w. those for which « that doesnot work »)

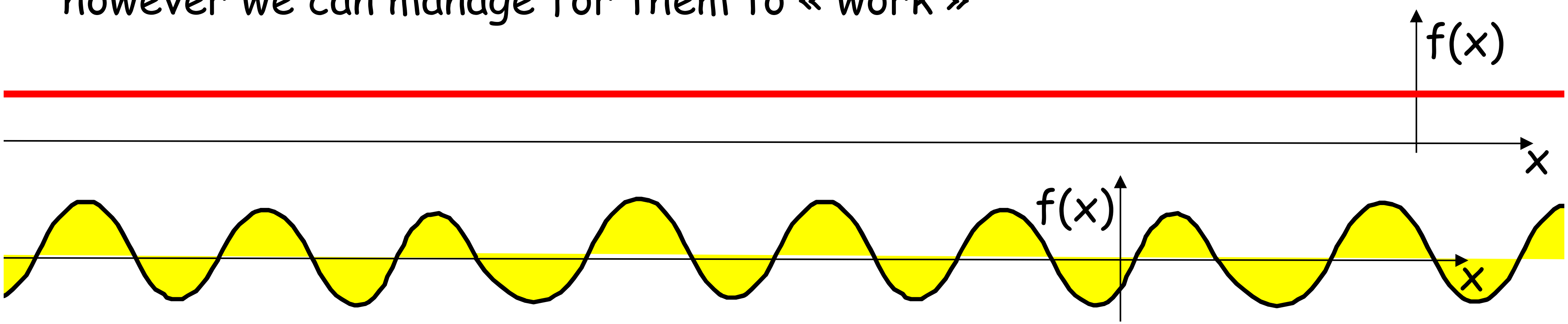




and so : constant = nasty ? **yes**

sine = nasty ? **yes**

however we can manage for them to « work »



supplementary note _2

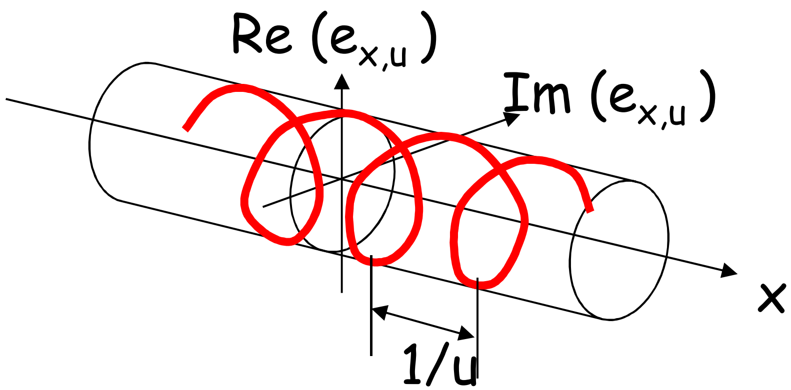
M

- the weighted sum of the sine functions defining f(x) is a continuous sum (integral over frequencies) extended to infinite

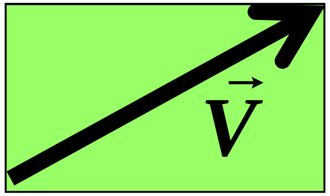
$$f(x) = \int_{-\infty}^{+\infty} weight(u).base\_component(x,u).du$$

- the components of the base are not simple sine functions but are complex exponential (helical shaped)

$$base\_component(x,u) = e(x,u) = \exp(\,i.2\pi.u.x)$$

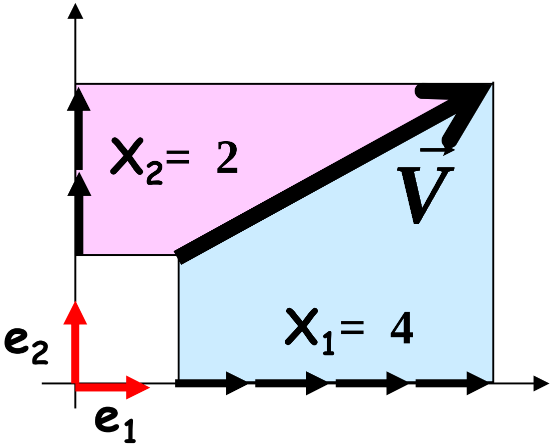


a **wild, candid** and disputable analogy with vectors



geometrical object

direction, orientation, length, origin



algebraic object built from a base (e1,e2)

two descriptions

$$\vec{V} = X_1 \,.\, \vec{e}_1 + X_2 \,.\, \vec{e}_2$$

direct

$$\vec{V} : \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} \qquad \vec{e}_1 : \begin{vmatrix} 1 \\ 0 \end{vmatrix} \qquad \vec{e}_2 : \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$

spectral

retrieving componants of the vector :

X_k ← scalar product

$$\vec{V} \,.\, \vec{e}_k = X_k$$

note : we may go to N componants

(and even we can imagine that N is infinite)

$$\vec{V} = \sum_{n=1}^{N} X_n \,.\, \vec{e}_n$$

sorry for the « rigueur »' !!!

$$\vec{V} = \sum_{n=-\infty}^{+\infty} X_n \,.\, \vec{e}_n$$

which relation with Fourier ?

M

Vectors :

componants X_n retrieved via the scalar product $\vec{V}.\vec{e_n}$

« TransFourized » :

fonctions « friendly" : the scalar product of f & g is :

$$< f , g > = \int f(x).g^{*}(x).dx$$

the scalar product of function f(x) & componant e(u,x) is :

$$\hat{f}(u) = < f , e_u > = \int f(x).e^{*}(u,x).dx$$

and we retrieve the componant linked to « u »:

$$\hat{f}(u) = \int f(x).e^{-i.2\pi.u.x}.dx$$

a bit like vectors $\vec{V}.\vec{e_n}$

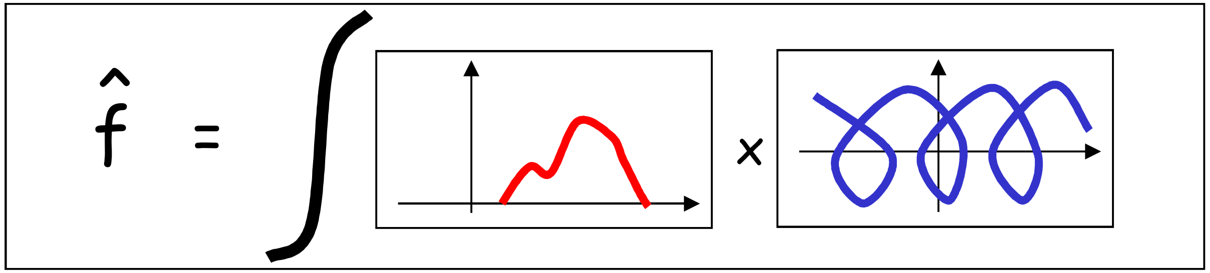
but what does mean all these horrible formulae ??

briefly

the FT tells what is the content in frequencies for « friendly » functions (and some others too)

algebra describes how to extract this contents

a less abstract approach : let's draw

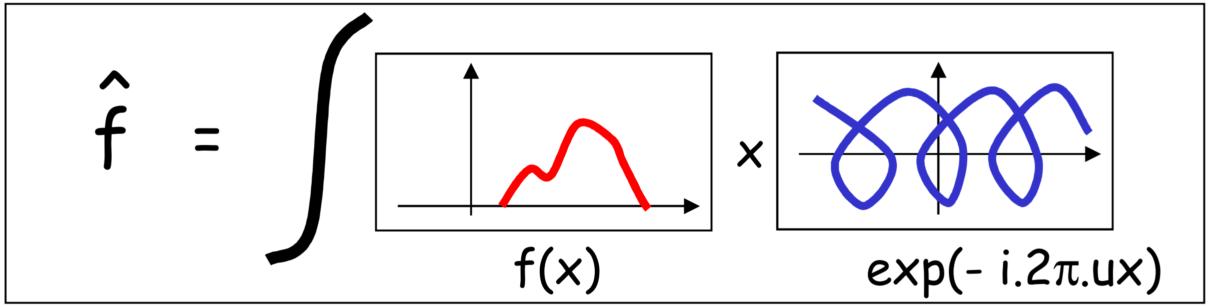


remember : the constant and the sine function do not match our algebra !

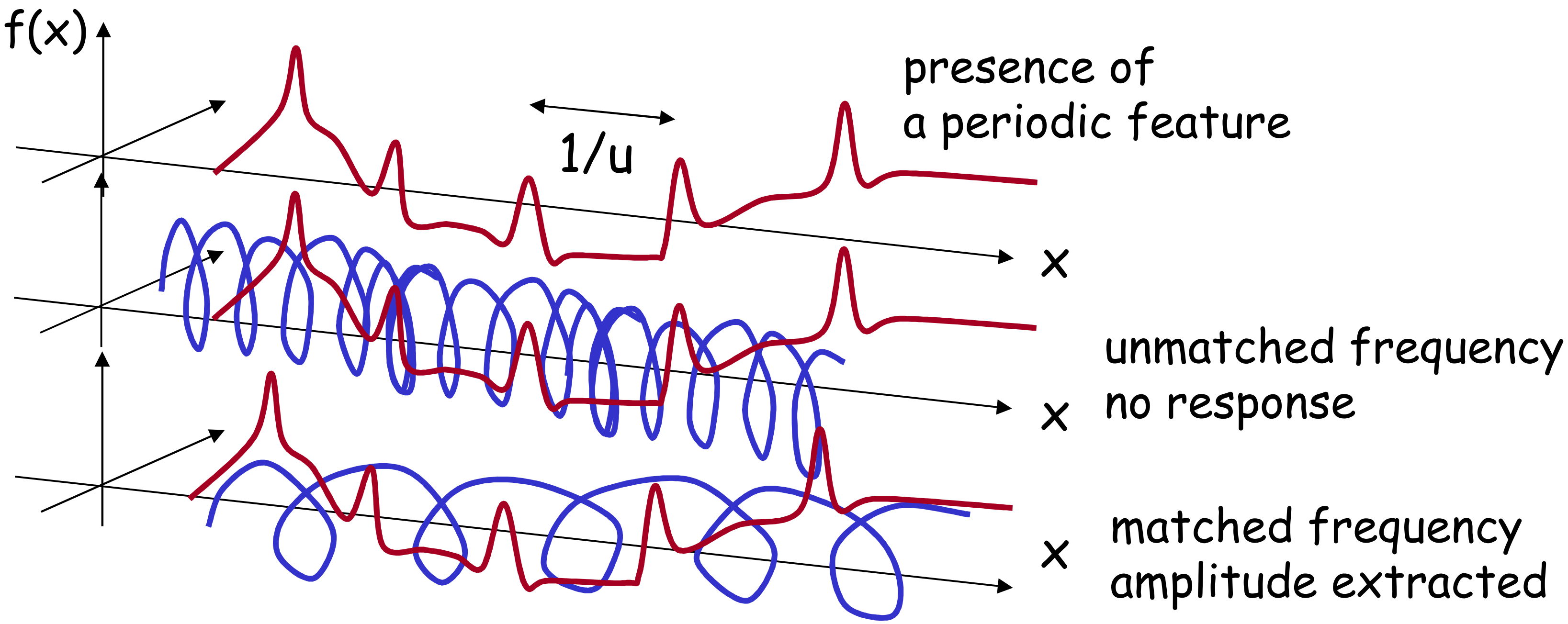
How to survive ? Dirac distribution !

practical view

a pictorial approach



FT can be seen as a "periodicity sensor" (period 1/u)
algebra yields the contribution of a periodic-like component



supplementary note _3

M

going from one description to the other

direct ➔ spectral : **FT**

$$\hat{f}(u) = \int f(x).e^{-i.2\pi.u.x}.dx$$

spectral ➔ direct : **inverse FT**

simply change the sign of the exponential

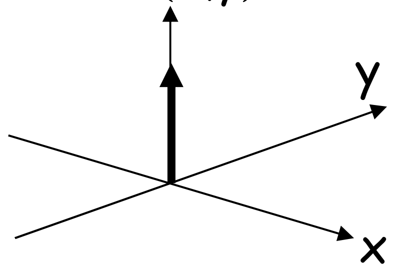
$$f(x) = \int \hat{f}(u).e^{+i.2\pi.u.x}.du$$

warning,
immediate difference with respect to vectors
actually here ^f(u) et f(x) may be seen as componants each to the other.
We will come back to this point later on

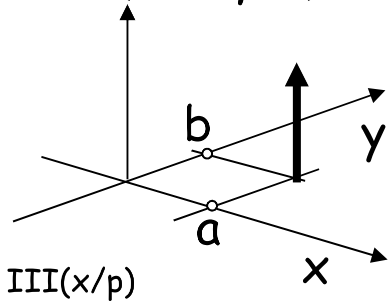
to tackle the situation more intuitively
we will make some « hand made » Fourier transforms,
but first, let's look at the Dirac distrtibution

the savior : Dirac world :

δ(x,y)



δ(x-a, y-b)

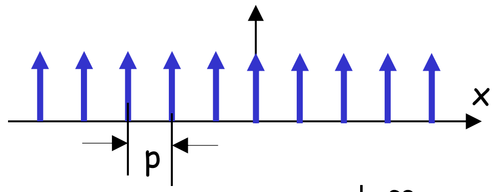


crucial relations

$$\int \delta(x).dx = \int \delta(x-a).dx = 1$$

$$f(x).\delta(x-a) = f(a).\delta(x-a)$$

$$\int f(x).\delta(x-a).dx = f(a)$$

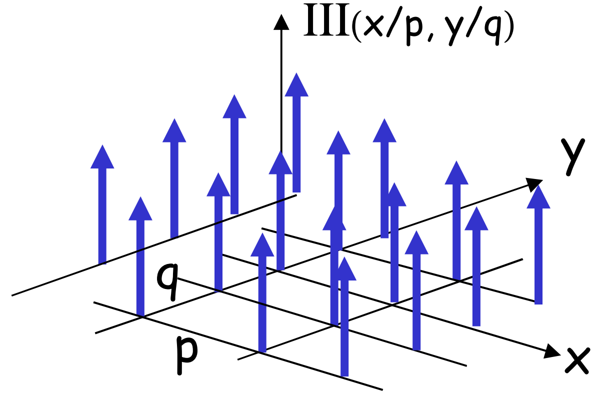


$$III(x/p) = \sum_{n=-\infty}^{+\infty} \delta(x-n.p)$$

$$1 \Leftrightarrow \delta(u)$$

$$\cos(2\pi.a.x) \Leftrightarrow \frac{1}{2}.[\delta(u-a)+\delta(u+a)]$$

$$\delta(x-a) \Leftrightarrow \exp(i.2\pi.u.a)$$

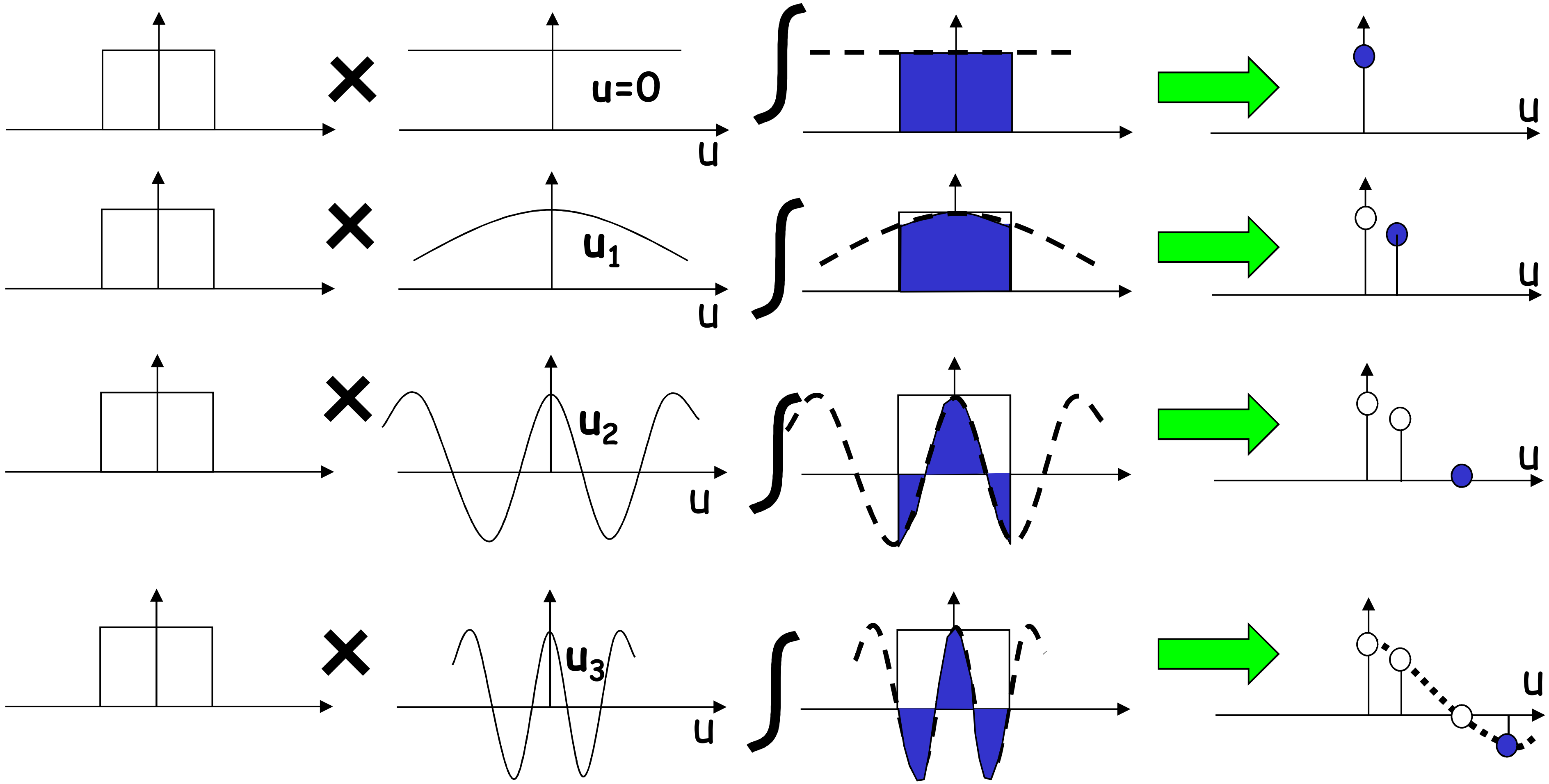


$$III(\frac{x}{p},\frac{y}{q}) = \sum_{n=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} \delta(x-n.p, y-k.q)$$

hand made transforming _ 1

rectangle, door, pulse

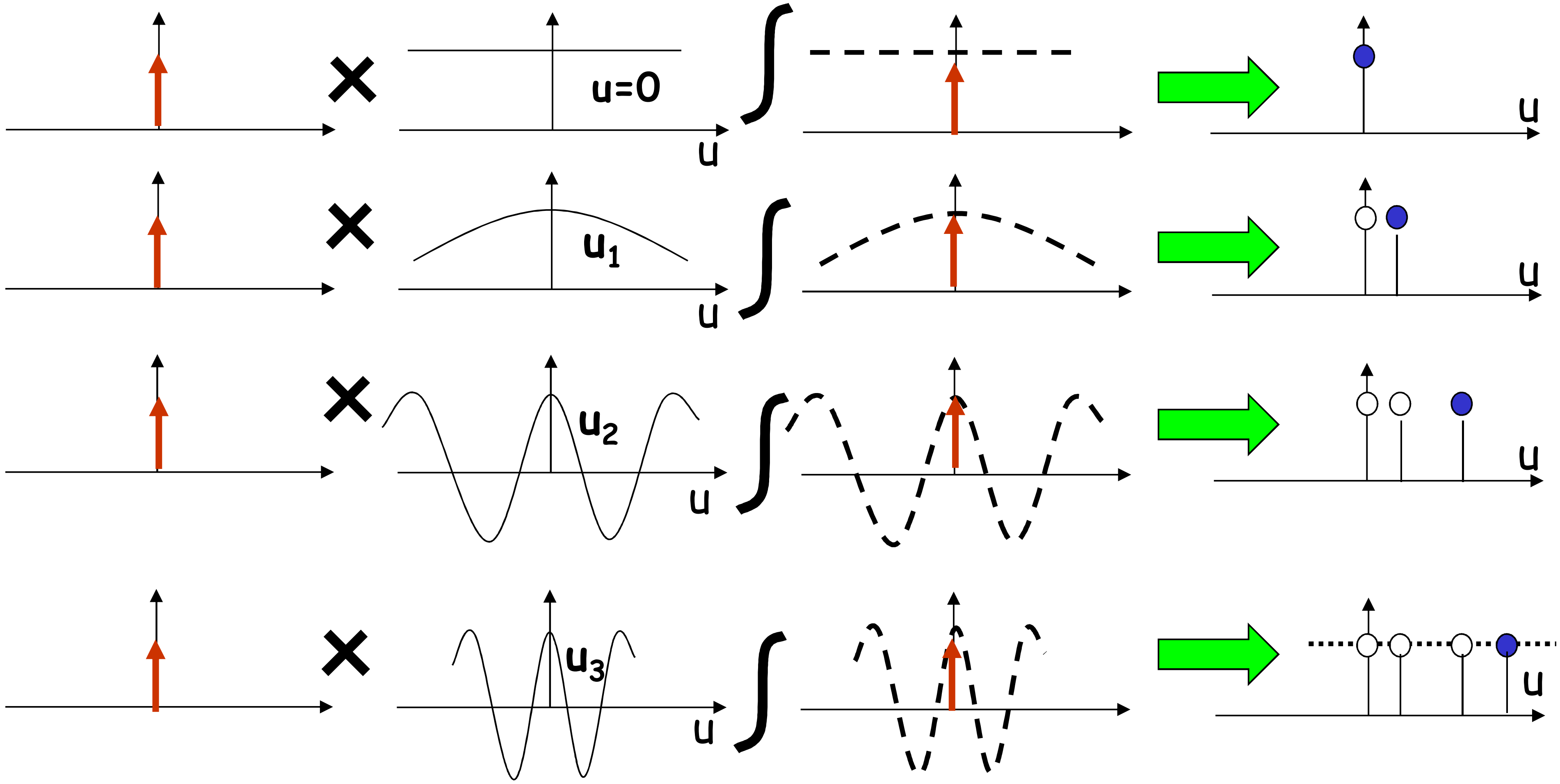
function | base function e(u,x) | integral => one value of TF
a spectrum's component



hand made transforming _2

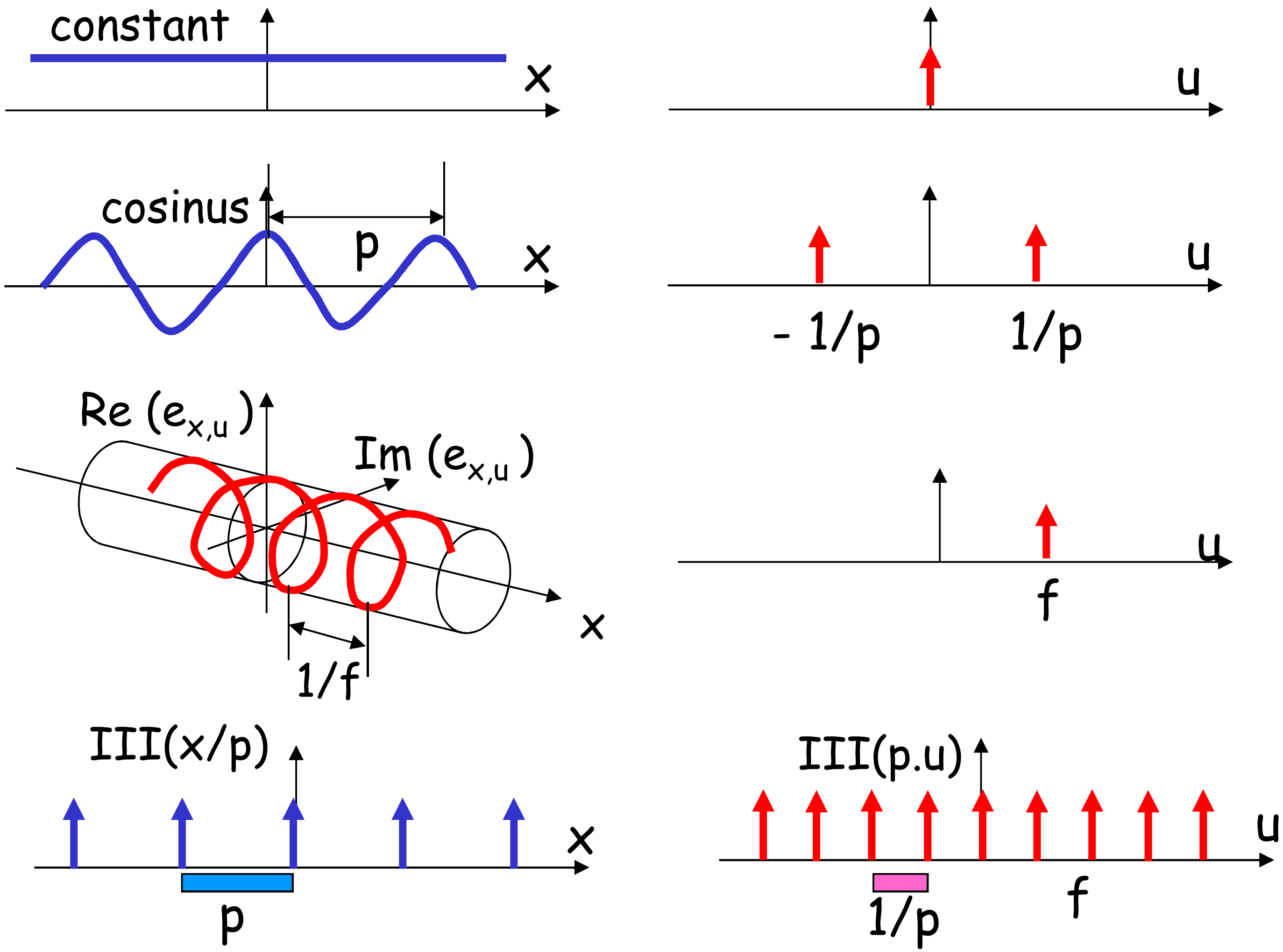
Dirac !

function base function e(u,x) integral => one value of TF
a spectrum's component



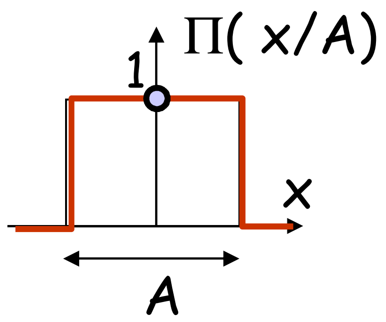
usual functions though rather pathological (but soon friendly)

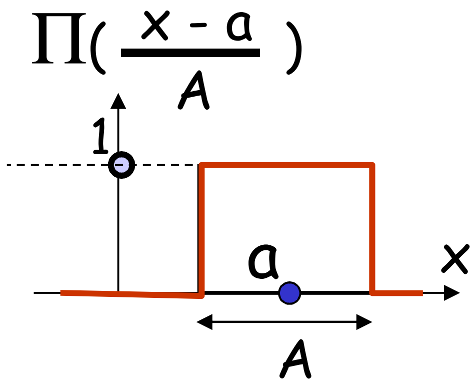
the definition integral does not converge : Dirac heals

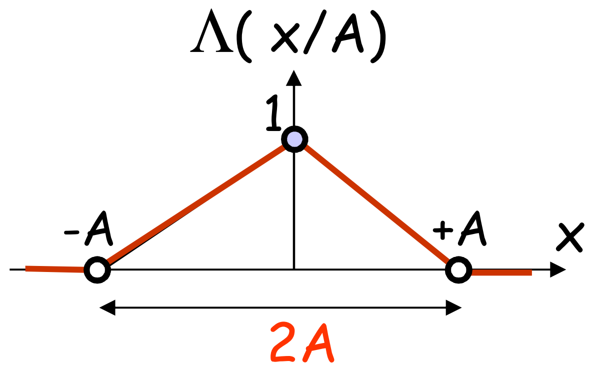


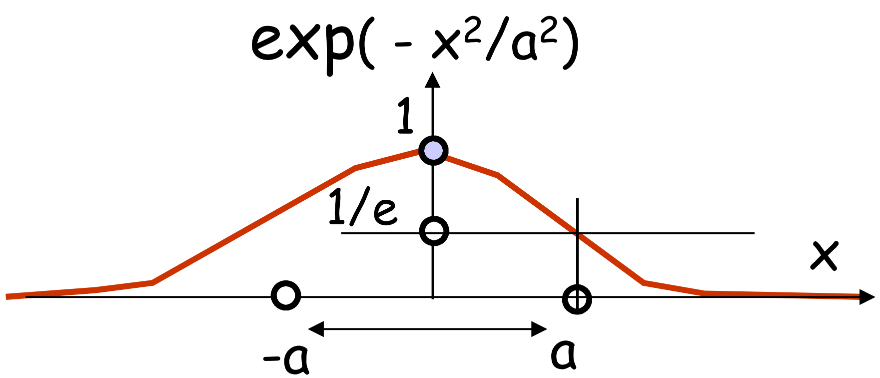
usual functions : notations and physionomy

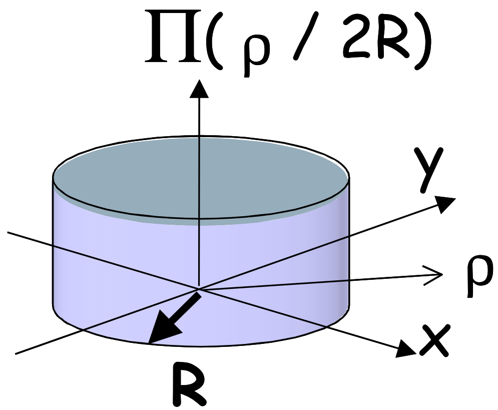
rectangle,pulse, door







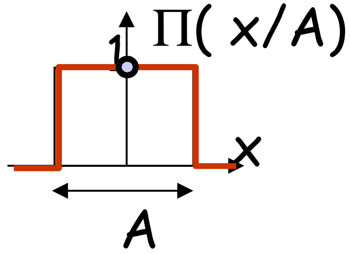




algebraic definition and notation for our usual functions

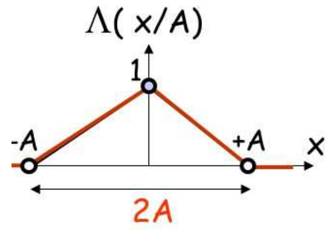
rectangle, pulse, door

$$\Pi\left(\frac{x}{A}\right) = \begin{cases} 1 \ \ if \ |x| < \ A/2 \\ 0 \ \ elsewise \end{cases}$$



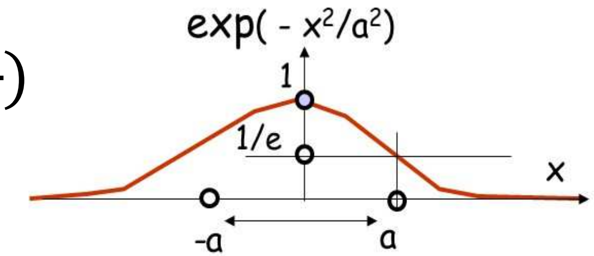
triangle or lambda function

$$\Lambda\left(\frac{x}{A}\right) = \begin{cases} 1 - |x| \ \ if \ |x| < A \\ 0 \ if \ |x| > A \end{cases}$$



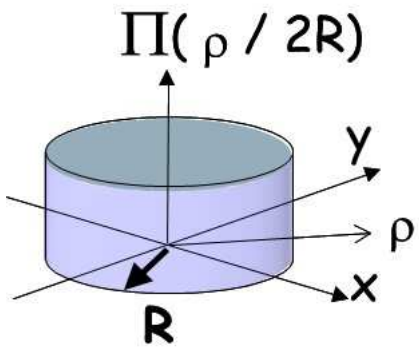
gaussian

$$G(x,a) = \exp\left(-\frac{x^2}{a^2}\right)$$



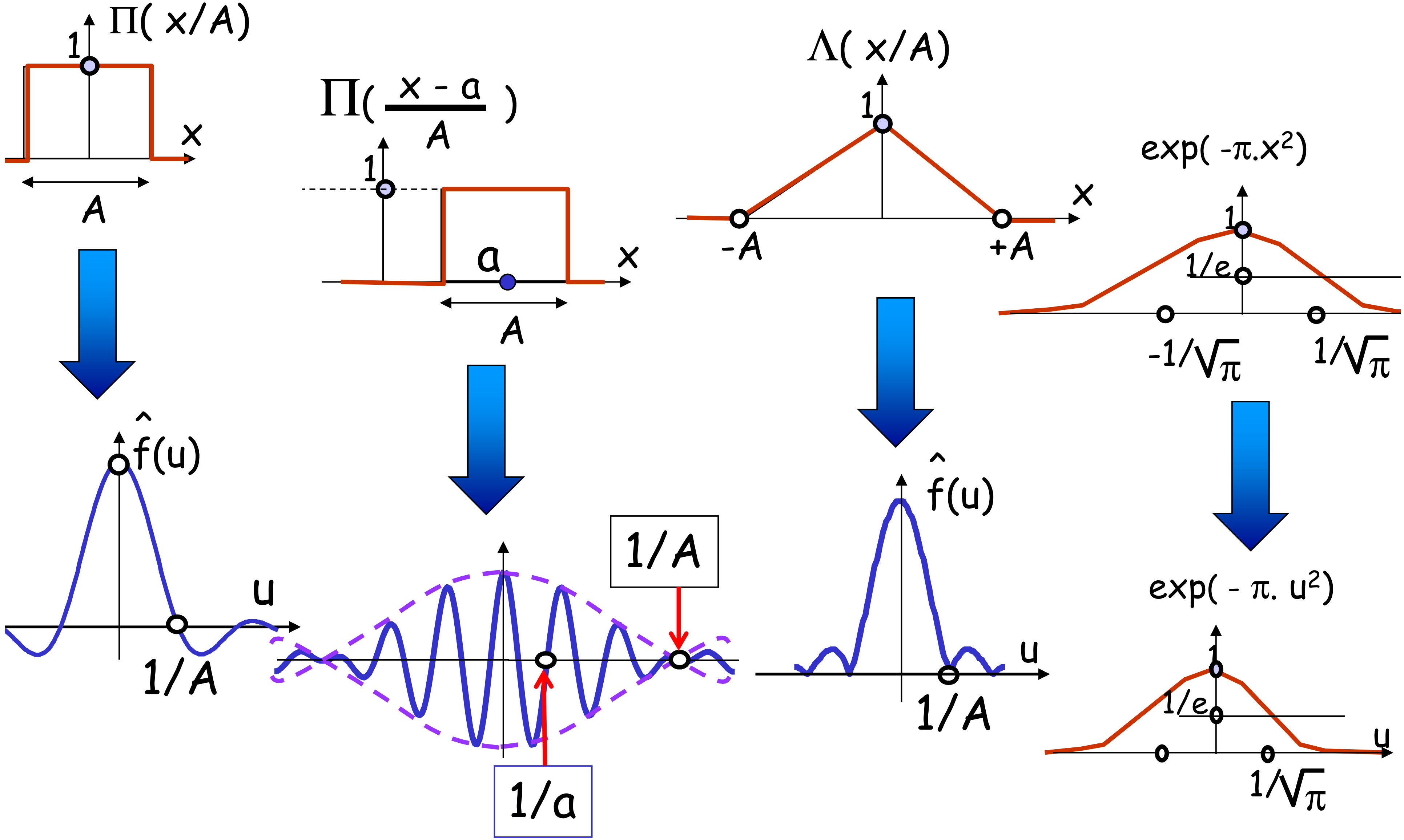
camembert or pil-box

$$\prod\left(\frac{\rho}{2R}\right) = \begin{cases} 1 \ if \ \rho < R \\ 0 \ \ elsewise \end{cases}$$



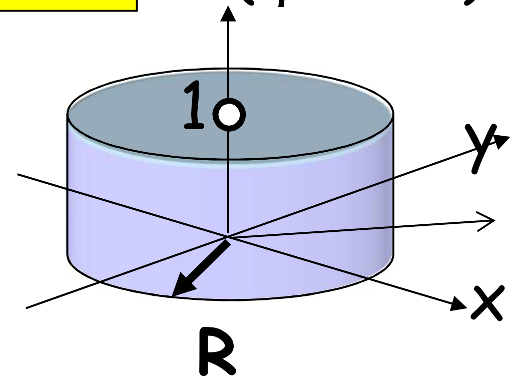
EXOS show physionomy of several modified functions

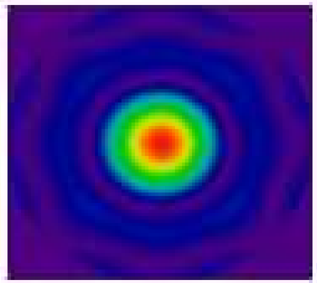
Fourier Transforms of usual functions _ 1

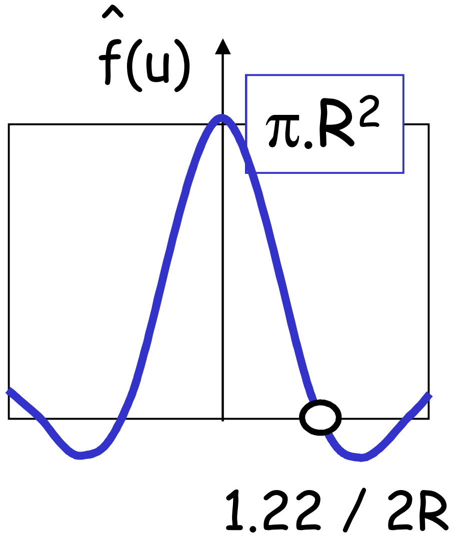


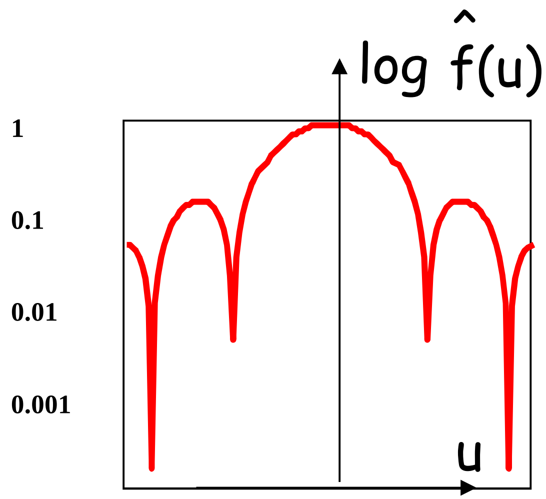
Fourier Transforms of usual functions _ 2

camembert





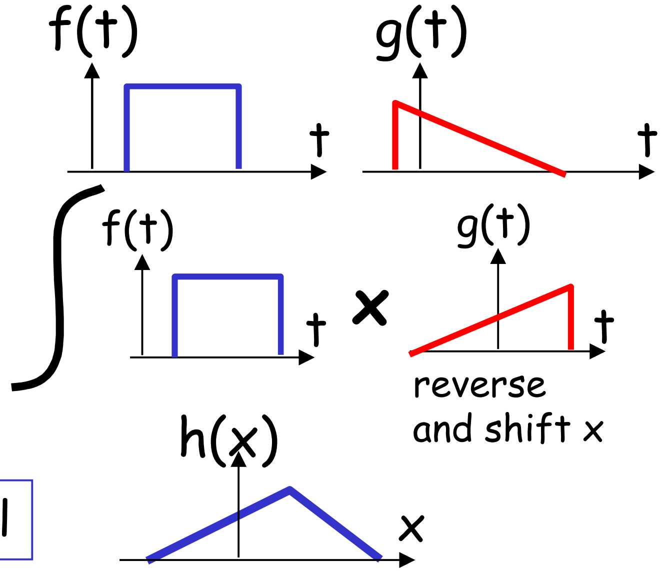




$$\Pi(\frac{\rho}{2R}) \quad \Rightarrow \quad \hat{\Pi}(q) = \left[\pi.R^2\right].\left[\frac{2.J_1(Z)}{Z}\right] = \left[\pi.R^2\right].Jinc(Z)$$

avec $Z = \pi.2R.q$

another tool : convolution



starting from two functions f(t) and g(t)
we define a new function h(x)
where x is the shifting of f versus g

$$h(x) = \int_{-\infty}^{+\infty} f(t).g(x - t).dt$$

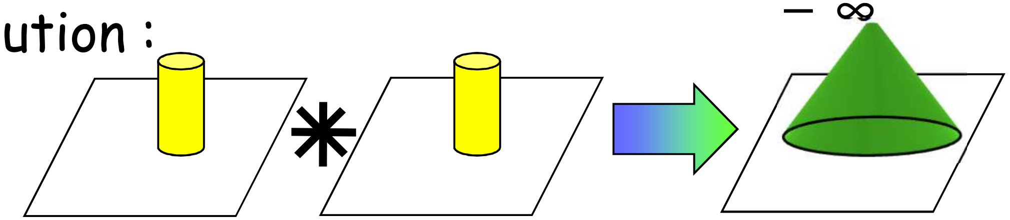
reversal

a (bad) current notation (physicists)

$$f(x) * g(x) = \int_{-\infty}^{+\infty} f(t).g(x - t).dt$$

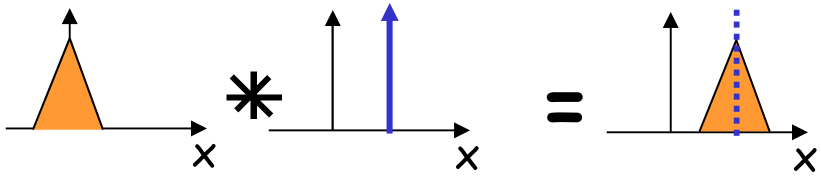
effects of convolution :

smoothing

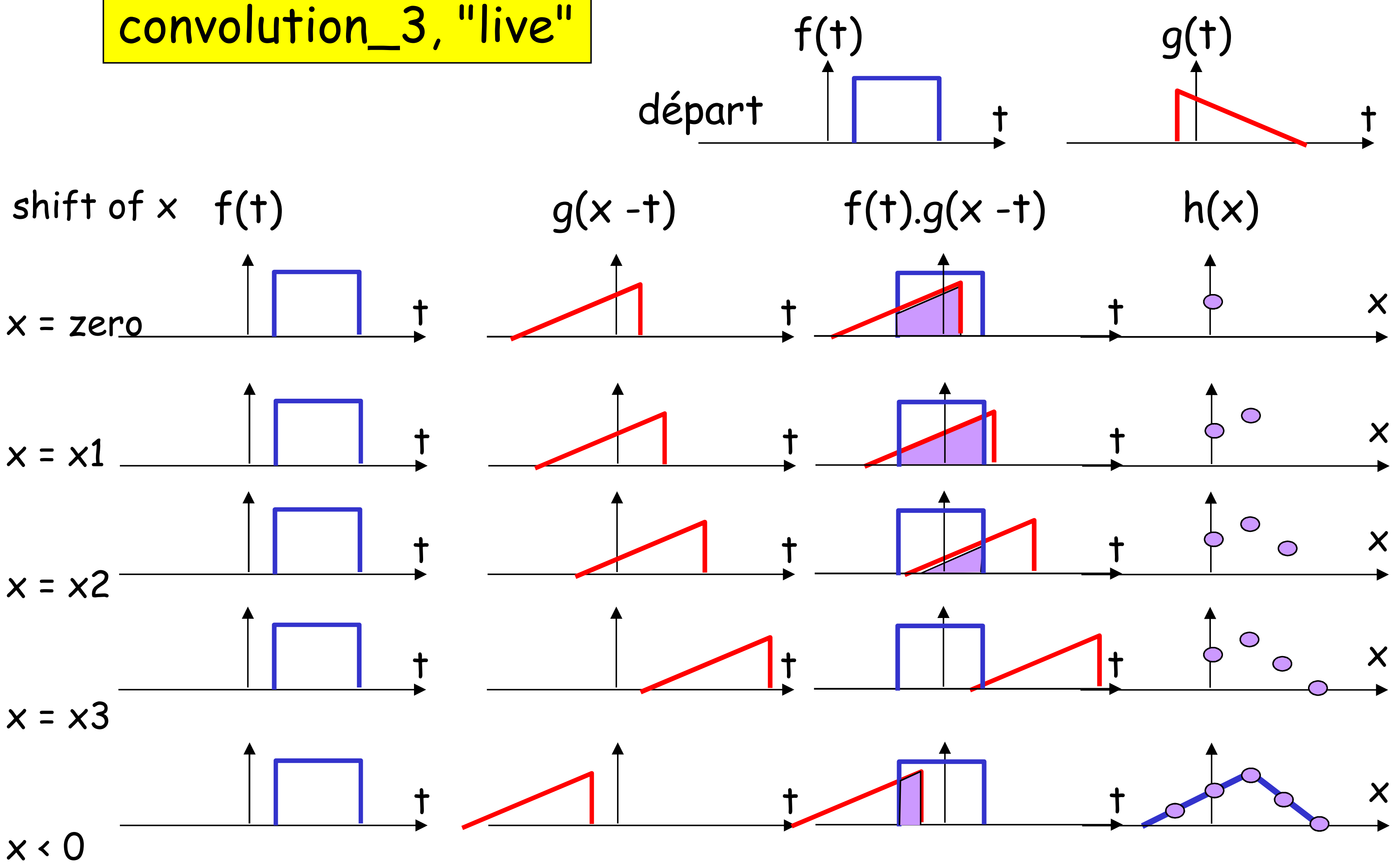


translating

$$f(x) * \delta(x-a) = f(x-a)$$

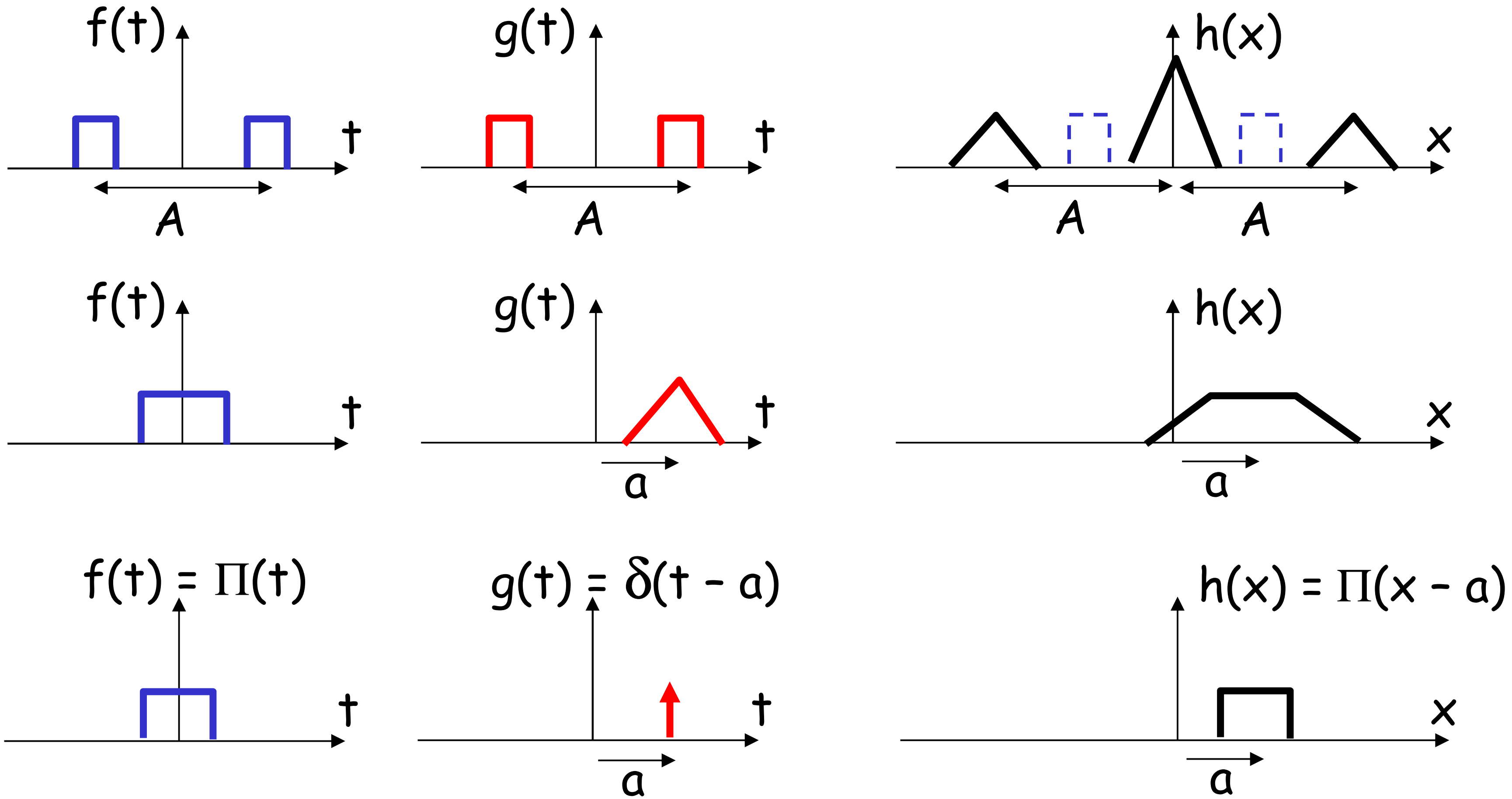


convolution_3, "live"



global result : convolution decreases stiffness, smoothing

convolution_3 visual training



$$\Pi(x) * \delta(x-a) = \Pi(x-a)$$

toolbox theorems for FT

starting point : $$f(x) \overset{FT}{\Leftrightarrow} \hat{f}(u)$$

translation theorem: $$f(x-a) \Leftrightarrow \hat{f}(u).e^{-i.2\pi.u.a}$$

similarity theorem $$f\left(\frac{x}{a}\right) \Leftrightarrow |a|\,\hat{f}(a.u)$$

convolution theorem $$f(x) * g(x) \Leftrightarrow \hat{f}(u).\hat{g}(u)$$

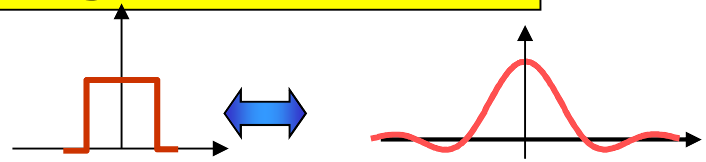
autocorrelation theorem $$|f(x)|^2 \Leftrightarrow \hat{f}(u) * \hat{f}(-u) = \hat{f}(u) \otimes \hat{f}(u)$$

very very very important

parseval (rayleigh) $$\int f(x).g*(x).dx = \int \hat{f}(u).\hat{g}*(u).du$$

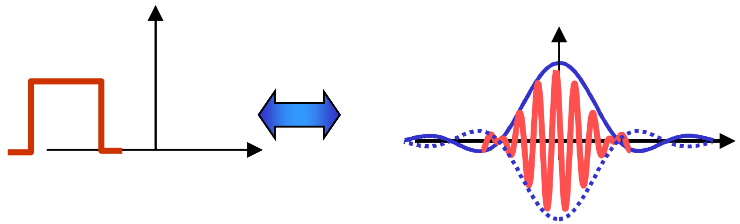
pictorial for theorerms with a rectangle ditribution

starting point : $f(x) \overset{TF}{\Leftrightarrow} \hat{f}(u)$



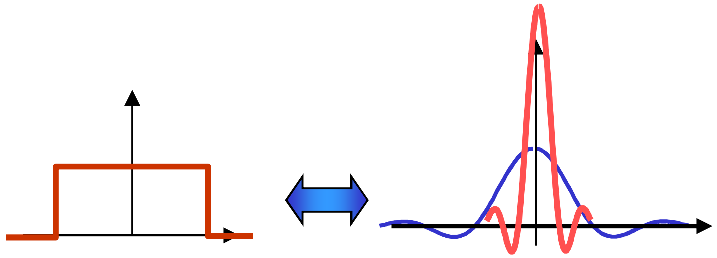
translation :

$f(x-a) \Leftrightarrow \hat{f}(u).e^{-i.2\pi.u.a}$



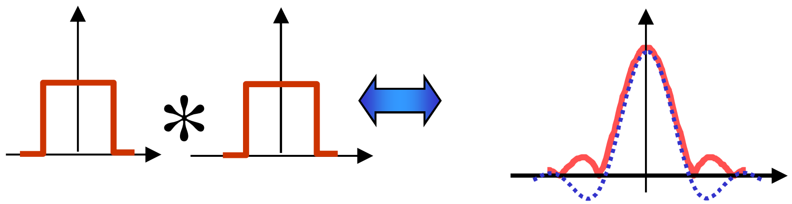
similarity

$f(\frac{x}{a}) \Leftrightarrow |a|.\hat{f}(a.u)$



convolution

$f(x) * g(x) \Leftrightarrow \hat{f}(u).\hat{g}(u)$

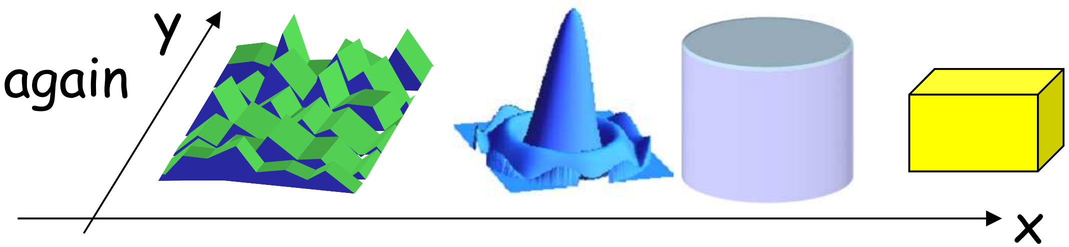


autocorrelation $|f(x)|^2 = f(u) \otimes g^*(u)$

parseval (rayleigh) $\int f(x).g^*(x).dx = \int \hat{f}(u).\hat{g}^*(u).du$

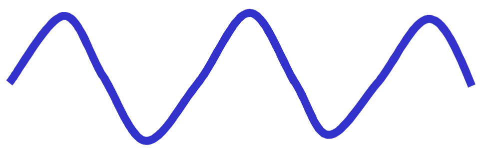
complication : introducing FT with 2 variables

"friendly" physical signals again



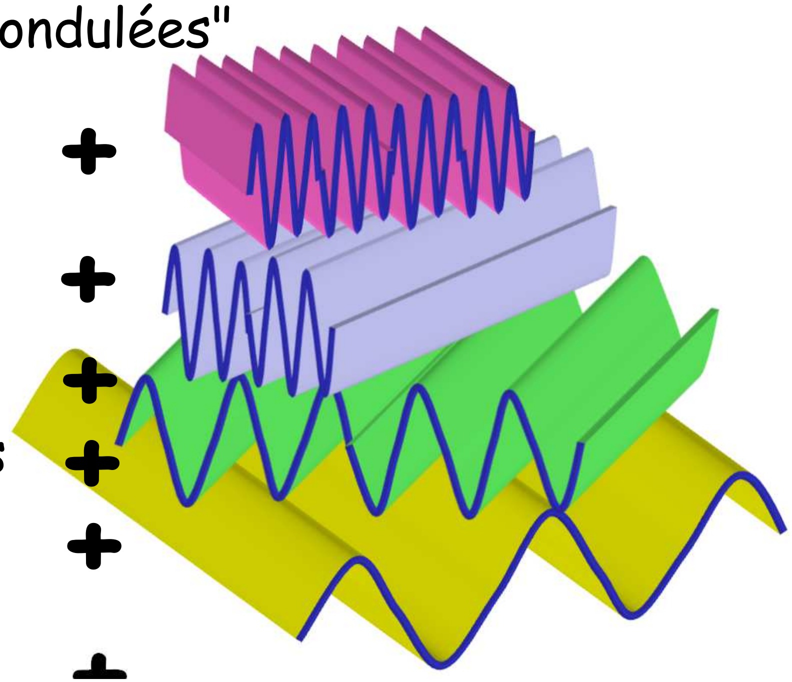
what is changing ??

the moduli of base functions
no longer are "sine lines"



moduli are now something like "tôles ondulées"
(but with negative parts)
having various periods, phases
and orientations

still, a given 2D-signal again is a
weighted sum of 2D- base functions



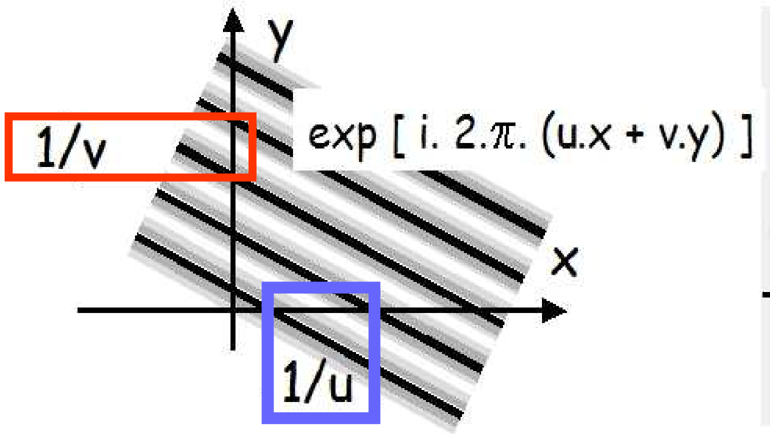
new algebra with two variables

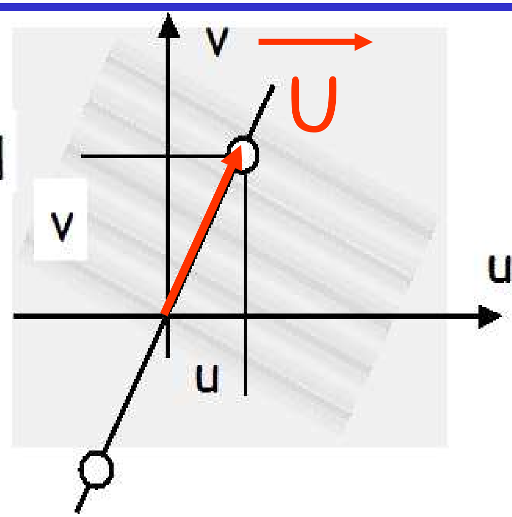
location of a given point of the object requires two coordinates (x and y) or a vector $\vec{X} : \begin{vmatrix} x \\ y \end{vmatrix}$

also, a given frequency is described by two "projected frequencies" "u" and "v", or a vector
(to account for orientation of the sine-like surface) $\vec{U} : \begin{vmatrix} u \\ v \end{vmatrix}$

then a given base function (varying over x and y) will convey (x,y,u,v) and writes

$$e(x ,y , u ,x) = e(\vec{X} ,\vec{U}) = exp(i .2\pi.\vec{X}.\vec{U}) = exp[i .2\pi.(u.x + v.y)]$$

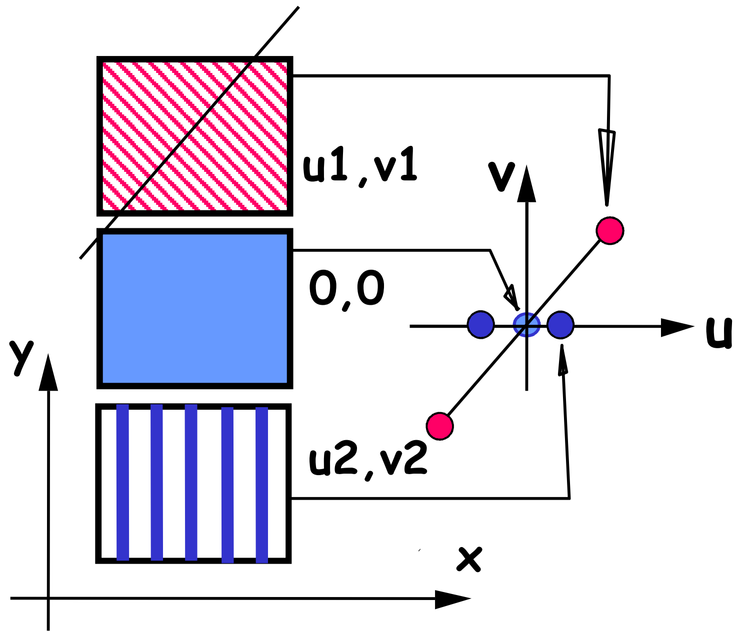




illustrations for 2 variables objects

A sine-like surface in (x,y) space

a couple of dirac-distributions
(FT of cosine)
in Fourier space
also named (u,v) plane



the "faster" the oscillation
the farther from origin the diracs (higher frequency)

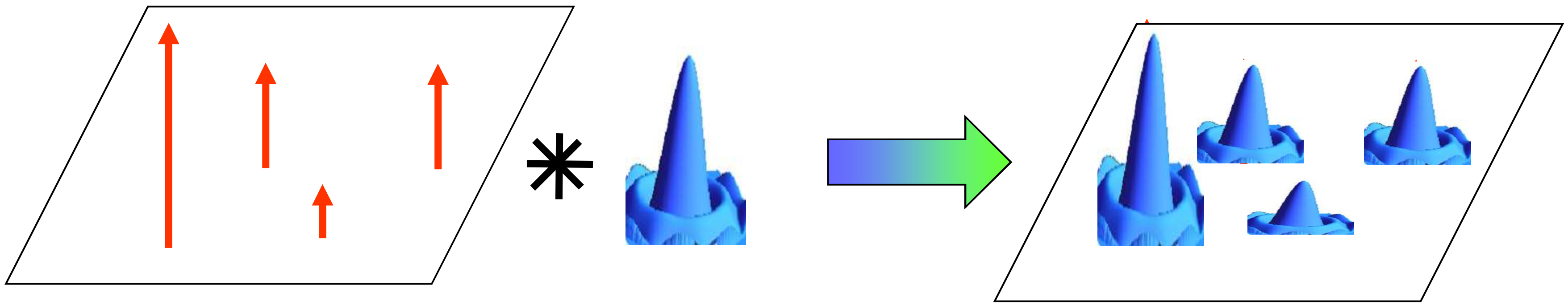
orientation of the couple is perpendicular to the wave crests

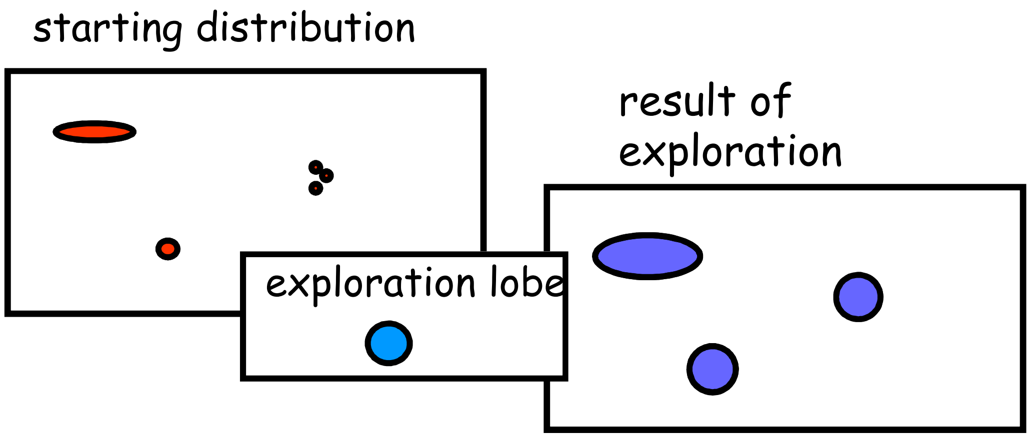
convolution_4 also with two variables

actors : f(t,z) and g(t,z) shifts : x and y

$$f(x,y) * g(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(t,z).g(x-t,y-z).dt.dz$$

illustration





this recalls something

we will come back to it again


need some rest
in quiet place !

limitations and subsequent needs

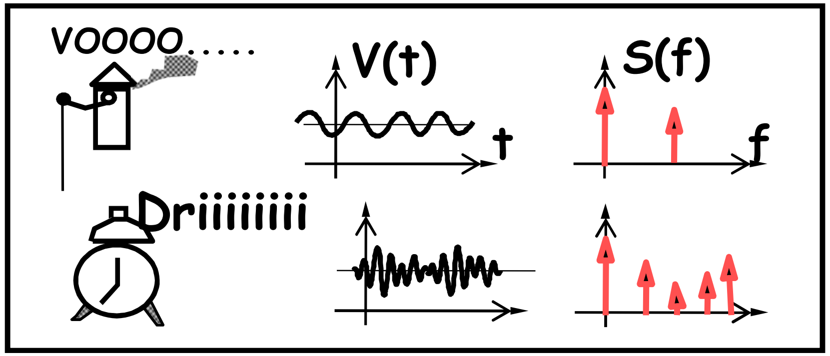
the first key :
spatial frequencies

frequencies , spectrum, in the familiar time domain

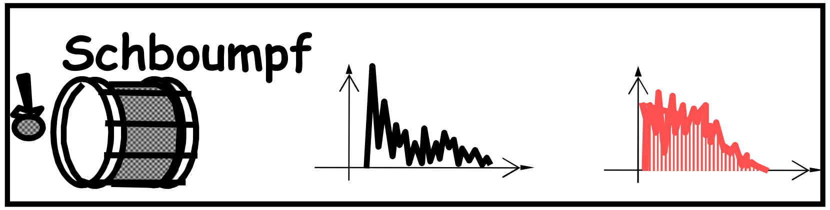
reminder : a physical signal can be described as a weighted sum of sinusoidal components (Fourier) of various frequencies

The set of weighting factors (Amplitude, frequency) is the spectrum of the signal

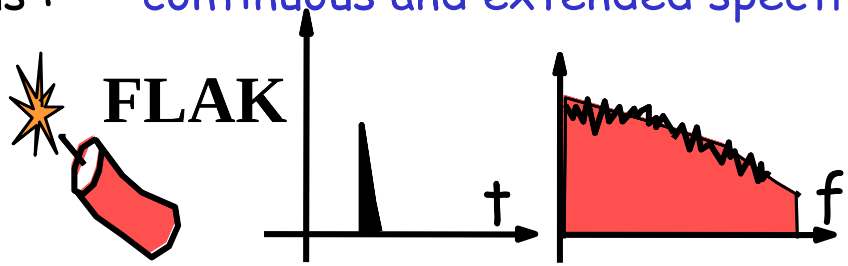
periodic signal : discrete spectrum



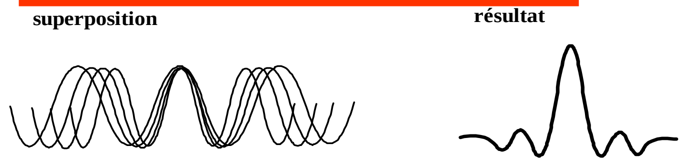
non-periodic : continuous spectrum



for very "narrow" signals, a lot of sinusoides needed and high frequencies
thus : continuous and extended spectrum



limiting case : Dirac



spatial frequencies

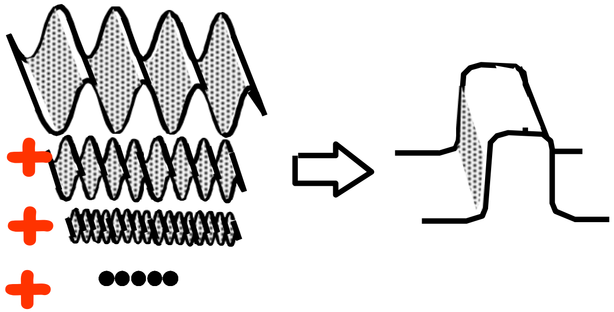
time domain : frequency = (1 / time) , Hz

spatial distribution (2-dim x & y)
spatialfrequency : vector (u,v) each component (1 / length)

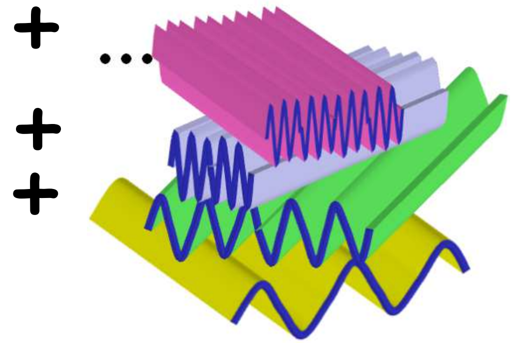
spatio-angular frequency : vector (u,v), each (1/angle) or (1/radian)

just recalling practical pictures :
"tole ondulée" or "sine surface" (with negative parts)

one direction modulated



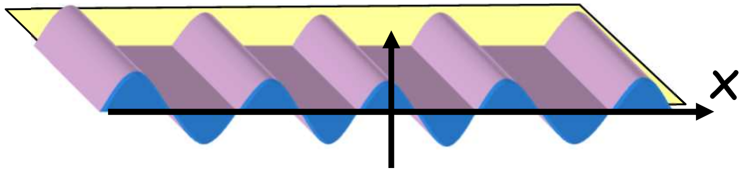
all directions modulated

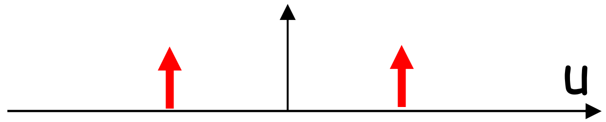


practical training : find spatial frequencies in the room !

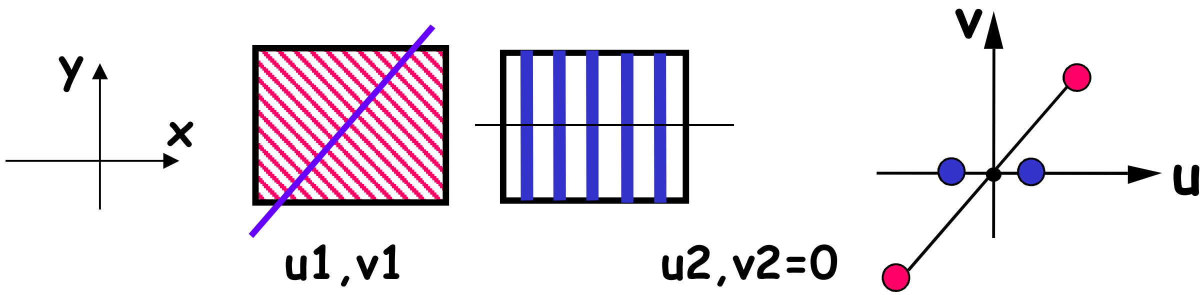
pictorial examples

"sine surface" (x,y) one direction --> couple of diracs in (u,v) plane (Fourier space)

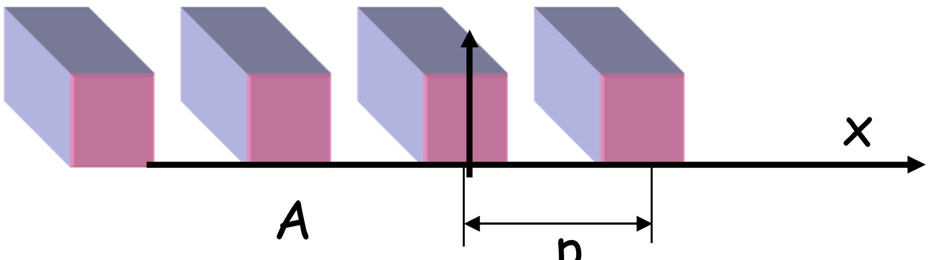


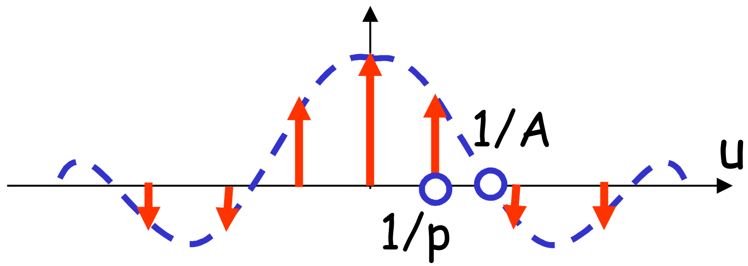


influence of orientation



replicated rectangle





algebra : EXO

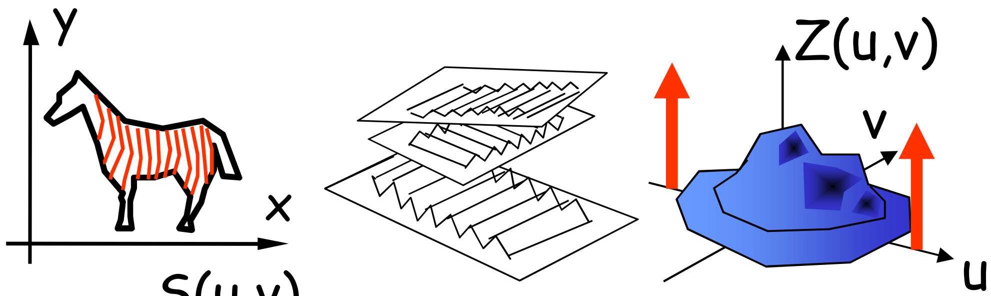
$$C(x) = \Pi\left(\frac{x}{A}\right) * III\left(\frac{x}{p}\right)$$

$$\hat{C}(u) \div \frac{\sin(\pi.A.u)}{\pi.A.u} . III(p.u)$$

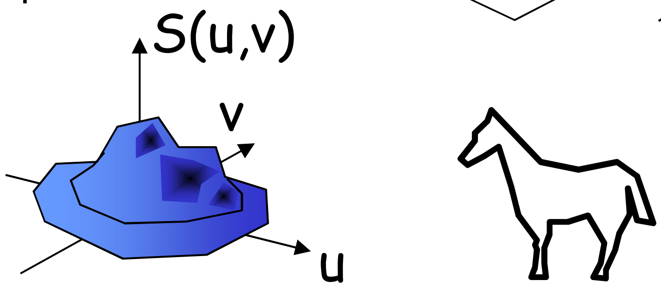
spatial frequencies : further training

change your zebra for a circus horse

Zebra =
horse + grid
(conspicuous spat. freq.)

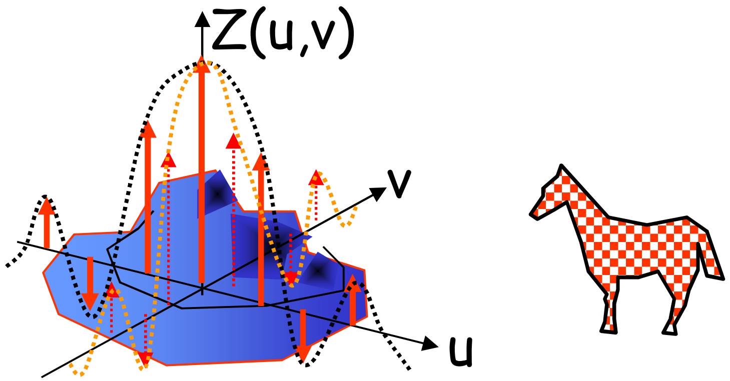


remove the grid



circus horse =
horse + double grid

just add
the needed frequencies



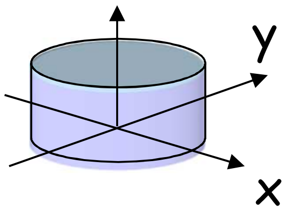
examples of spatial distributions and spatial spectra _ 1

spatial distribution: $O(x,y)$ or $O(\alpha,\beta)$

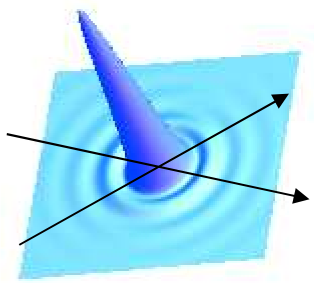
spatial spectrum (2-dim Fourier Transform) $\hat{O}(u,v)$

WARNING ! : spatial spectrum is a complex function
only modulus displayed here

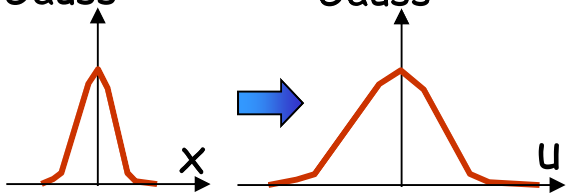
camembert



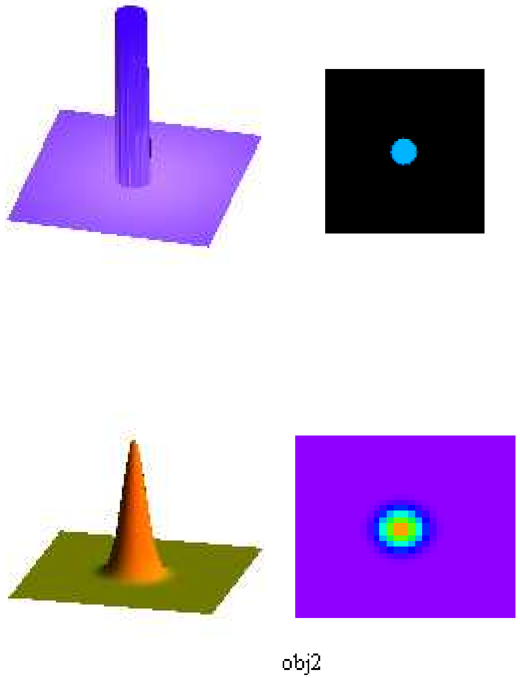
bessel



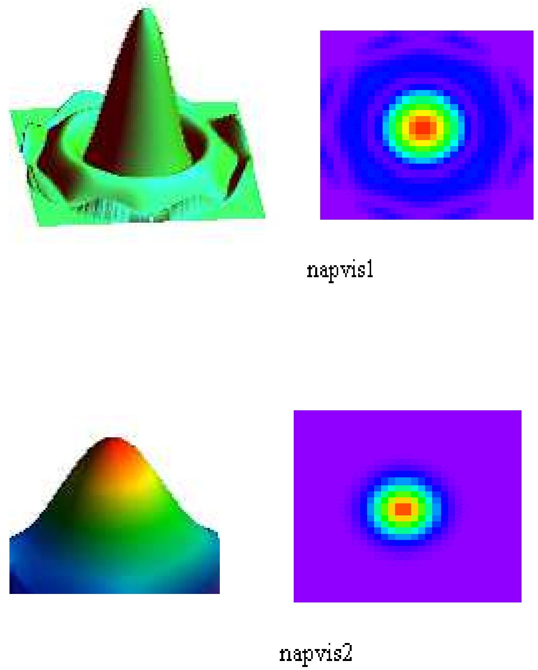
Gauss Gauss



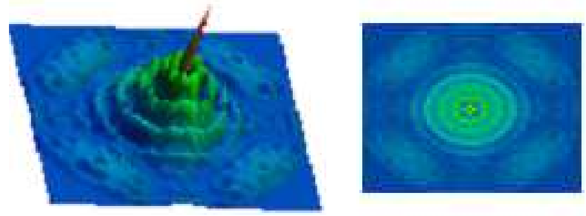
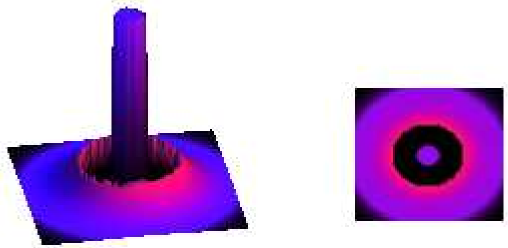
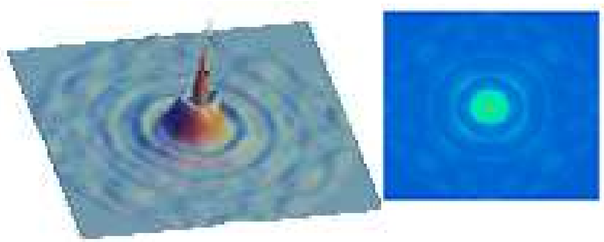
distributions



spectra(modulus)



examples of spatial distributions and spatial spectra _2

	distributions	spectra (modulus)
narrow cylinder + co-centric corona		
cylinder + co-centric hollowed gaussian		

examples of spatial distributions and spatial spectra _3

couple of diracs
unequal strength

$$O(x,y)=\delta(x,y)+h.\delta(x-\rho,y)$$

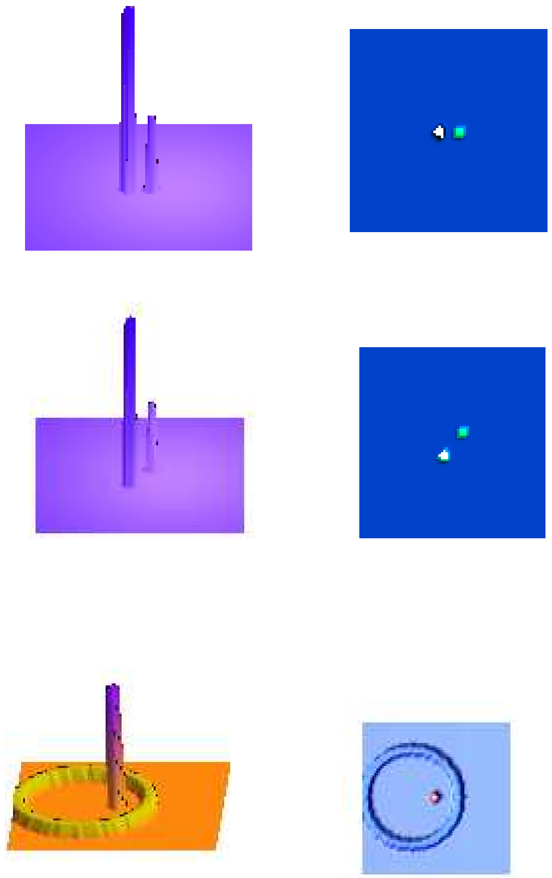
dirac
unequal strength
oblique orientation

now with

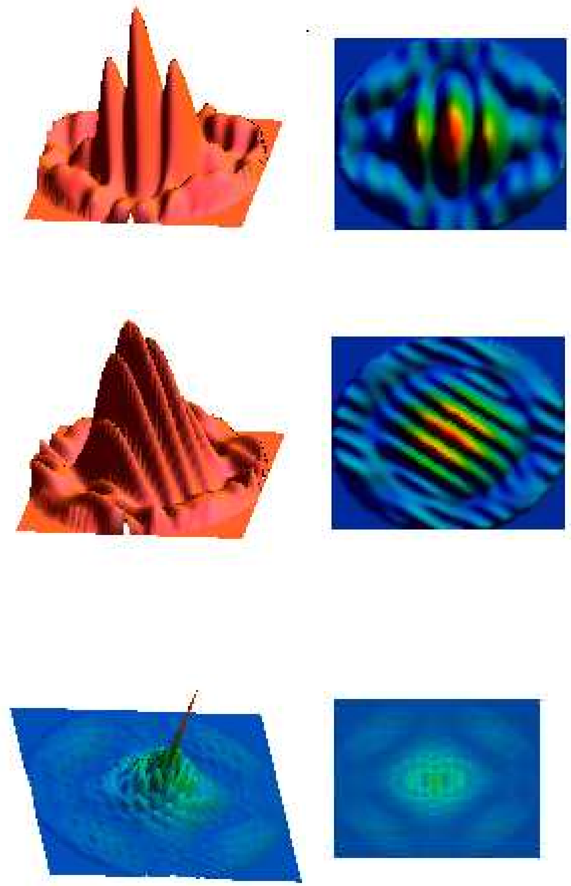
$$h.\delta(x-\rho,y-\eta)$$

cylinder +
off-centered
corona

distributions



spectra (modulus)



all that pictures can be recasted with "source" or "object" instead of "distribution"

living in Fourier Space

distribution	spectrum (module)
narrow extension	large extension
composite	composite (linearity of FT)
A narrow + B large	$\hat{A}$ *large* + $\hat{B}$ *narrow* but take care of separation of centers (complex exponential)
stiff edges, sharp angles	strong high frequencies
smooth shape	faint high frequencies (Gauss)
dissymetry	spatial modulation, phase effect (complex exponential, fringes)

to see again later

coming back to our suggested technical solution

conventional imaging does not work !

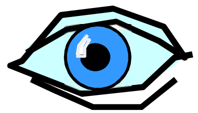
a new approach is needed to describe the brightness distribution or at least to obtain some of its parameters

answer is aperture synthesis which is based on **interferometry**

the idea behind :
fetch spatial information in Fourier Space !

in other words : determine **spatial spectrum** and then, come back as far as possible to the angular intensity distribution of the source

conventional imaging : directly provides a 2-dim representation (feeding a camera)



with aperture synthesis the imaging process requires specific methods for observation and computation