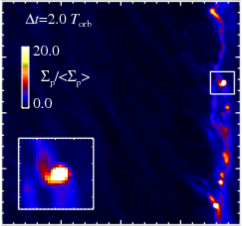
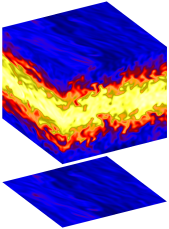
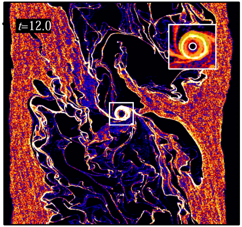
Planetesimal formation by self-gravity



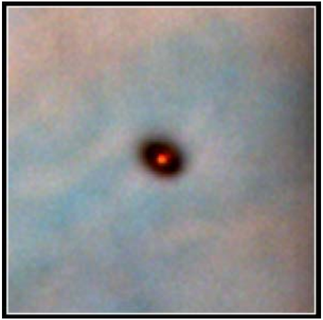
"Chronology of solar system formation V: the first solids and the first planetesimals"

Les Houches winter school

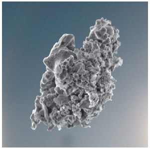
February 2013

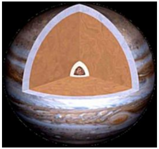
Anders Johansen (Lund Observatory)

Conditions for planet formation



- All young stars are orbited by protoplanetary discs
- Disc masses of 10^{-4}–10^{-1} $M_\odot$
- Disc life-times of 1–3 million years



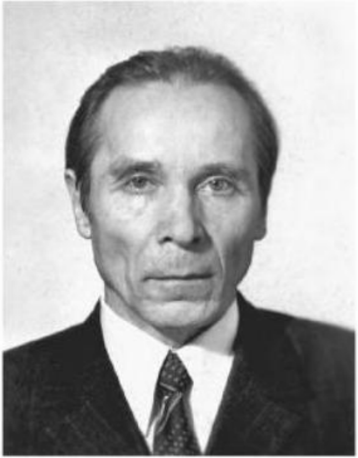


Planet formation paradigm

Planetesimal hypothesis:

Planets form in protoplanetary discs around young stars from dust and ice grains that collide to form ever larger bodies

- Viktor Safronov (1917-1999): "father" of the planetesimal hypothesis
- "Evolution of the Protoplanetary Cloud and Formation of the Earth and the Planets" (1969, translated from Russian)

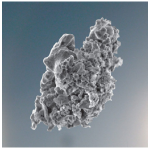


The three steps of planet formation

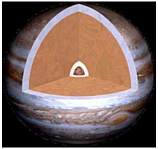
Planetesimal hypothesis of Safronov 1969:

Planets form in protoplanetary discs around young stars from dust and ice grains that collide to form ever larger bodies

1. Dust to planetesimals
   μm $\rightarrow$ km: contact forces during collision lead to sticking
2. Planetesimals to protoplanets
   km $\rightarrow$ 1,000 km: gravity (run-away accretion)
3. Protoplanets to planets
   Gas giants: 10 $M_\oplus$ core accretes gas ($< 10^6$ years)
   Terrestrial planets: protoplanets collide (10^7–10^8 years)

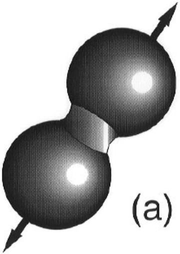




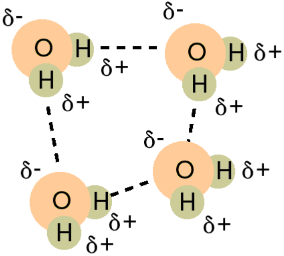


Sticking

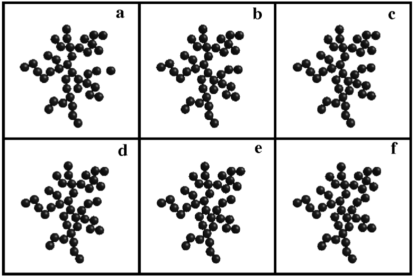
- Colliding particle stick by the same forces that keep solids together (van der Waals forces such as dipole-dipole attraction)

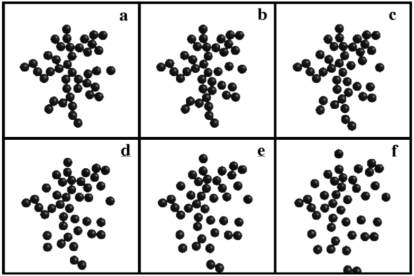


(a)



Dust experiments

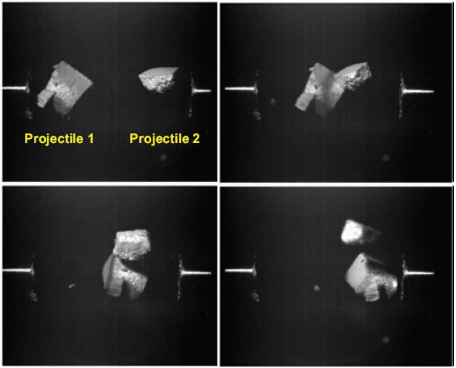




- Use μm-sized *monomers as building blocks* in particle simulation with contact forces
- Left plot: collision between aggregate and monomer at 20 m/s (sticking)
- Right plot: collision between aggregate and monomer at 200 m/s (fragmentation)
- Computer simulations from Dominik & Tielens (1997)

Laboratory experiments

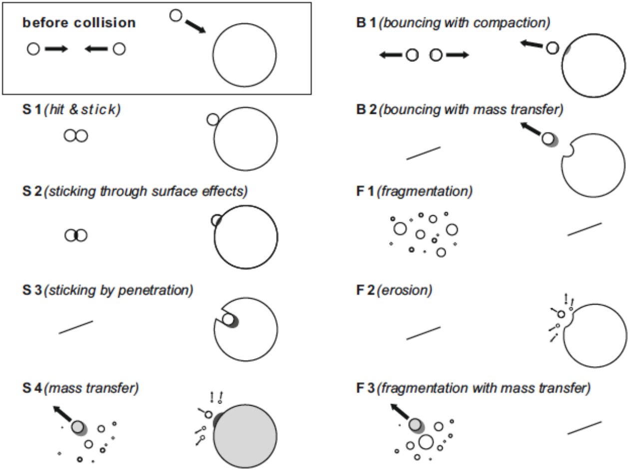
- Laboratory experiments used to probe sticking, bouncing and shattering of particles (labs e.g. in Braunschweig and Münster)
- Collisions between equal-sized macroscopic particles lead mostly to bouncing:



- From Blum & Wurm (2008)

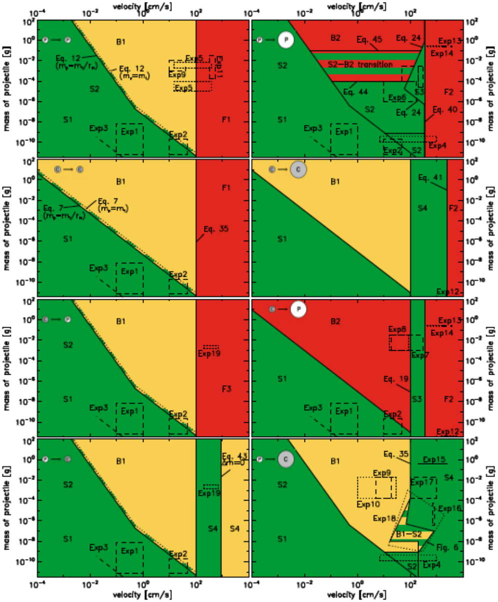
Collision regimes

- Güttler et al. (2010) compiled experimental results for collision outcomes with different particle sizes, porosities and speeds



Collision outcomes

- Güttler et al. (2010):
- Generally sticking below 1 m/s and bouncing or shattering above 1 m/s
- Sticking may be possible at higher speeds if a small impactor hits a large target



Drag force

Gas accelerates solid particles through drag force:

$$\frac{\partial \mathbf{v}}{\partial t} = \ldots - \frac{1}{\tau_\mathrm{f}}(\mathbf{v} - \mathbf{u})$$

Particle velocity

Gas velocity

In the Epstein drag force regime, when the particle is much smaller than the mean free path of the gas molecules, the friction time is

$$\tau_\mathrm{f} = \frac{R\rho_\bullet}{c_\mathrm{s}\rho_\mathrm{g}}$$

R: Particle radius
$\rho_\bullet$: Material density
c_s: Sound speed
ρ_g: Gas density

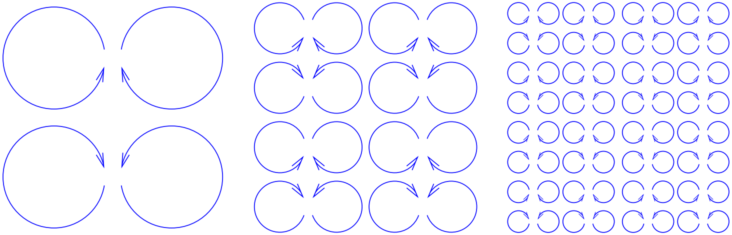
Important nondimensional parameter in protoplanetary discs:

$$\Omega_\mathrm{K}\tau_\mathrm{f} \text{ (\textit{Stokes number})}$$

Size of $\mathrm{St} = 1$ particles scales in MMSN as $R_1 \approx 30\,\mathrm{cm}\,[r/(5\,\mathrm{AU})]^{-1.5}$

Turbulence

Lewis Fry Richardson about turbulence: *Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity*

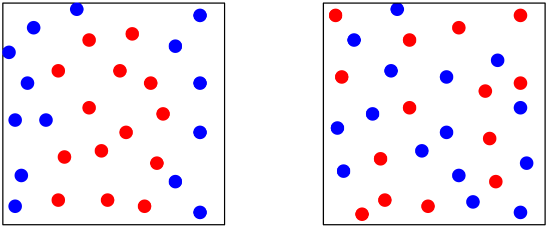


The general concept of turbulence:

- Energy injected at scale L
- Eddies break into smaller eddies which break into smaller eddies etc.
- Energy present at all scales
- Molecular viscosity dissipates energy at *Kolmogorov scale* ($\sim$ 1 km in protoplanetary discs)

Turbulent mixing

- Take a box with equally many red and blue balls
- Separate the red balls from the blue balls
- Let all balls random walk $N \gg 1$ steps



$\Rightarrow$ The two colours mix completely

$\Rightarrow$ Turbulence evens out concentrations (*turbulent diffusion*)

$\Rightarrow$ Turbulence evens out velocity differences (*turbulent viscosity*)

Turbulent viscosity

Major problem: viscous time-scale is much longer than a Hubble time

$$t_{\rm visc} = \frac{R^2}{\nu}$$

von Weizsäcker 1948:
Replace molecular viscosity ν by turbulent viscosity $\nu_{\rm t}$

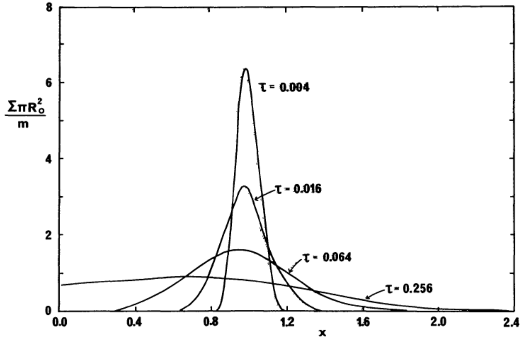
Shakura & Sunyaev 1973:
Assume that typical eddies have scale $\ell \sim \sqrt{\alpha} H$ and velocity $v_{\rm t} \sim \sqrt{\alpha} c_{\rm s}$

Famous α prescription of Shakura Sunyav 1973:

$$\nu_{\rm t} \sim \ell \times v_{\rm t} \sim \alpha H c_{\rm s}$$

Evolution of viscous discs

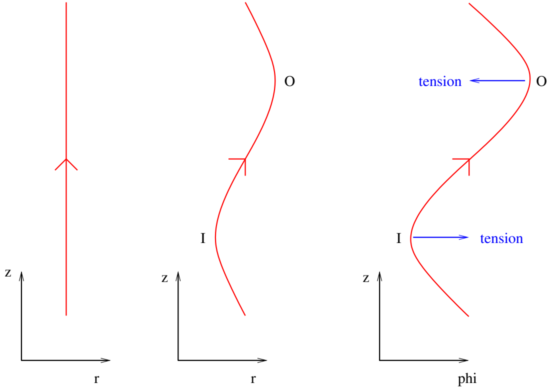
$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}\left(\nu \Sigma R^{1/2}\right)\right]$$



Time evolution of narrow ring of material with constant ν (from Pringle 1981). The equilibrium mass flux is

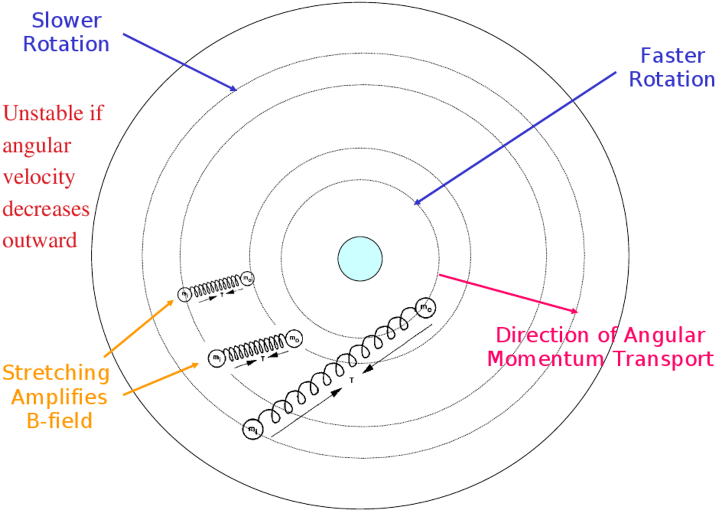
$$\dot{M} = 3\pi \Sigma \nu$$

Magnetorotational instability



1. Magnetic field line threading the disc
2. Excited mode (k_z only)
3. Interior point I orbits faster than exterior point O, amplifying B_r *and* the magnetic tension

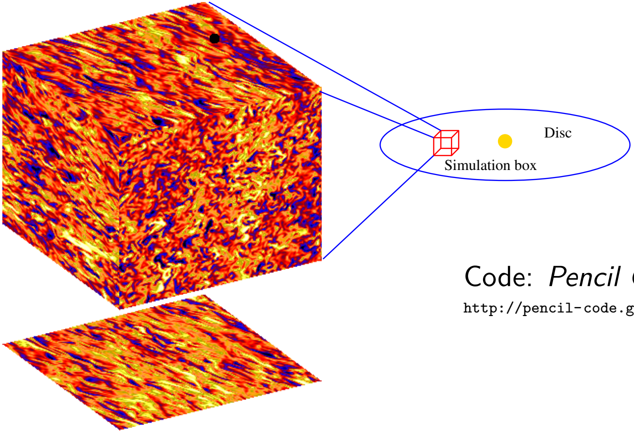
Magnetorotational instability sketch



- MRI derived in its modern form by Balbus & Hawley (1991)

Magnetorotational turbulence

- To model the non-linear evolution of the MRI, we need to solve the full set of dynamical equations numerically in 3-D



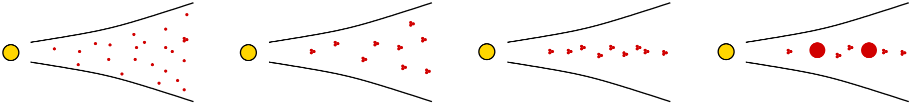
Code: *Pencil Code*

`http://pencil-code.googlecode.com`

- Measured viscosity $\alpha \sim 10^{-4}$–10^{-1} *(e.g. Hawley et al. 1995; Brandenburg et al. 1995)*

$\Rightarrow$ Turbulence caused by MRI can explain accretion onto young stars

Sedimentation



- Dust grains coagulate and gradually decouple from the gas
- Sediment to form a thin mid-plane layer in the disc
- Planetesimals form by continued coagulation or self-gravity (or combination) in dense mid-plane layer

HOWEVER:
MRI-driven turbulence very efficient at diffusing dust

Diffusion-sedimentation equilibrium

Diffusion-sedimentation equilibrium:

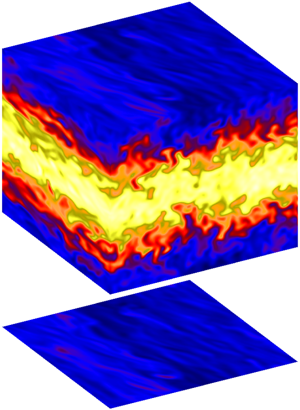
$$\frac{H_{\rm dust}}{H_{\rm gas}} = \sqrt{\frac{\delta_{\rm t}}{\Omega_{\rm K}\tau_{\rm f}}}$$

$H_{\rm dust}$ = scale height of dust-to-gas ratio

$H_{\rm gas}$ = scale height of gas

$\delta_{\rm t}$ = turbulent diffusion coefficient, like α-value

$\Omega_{\rm K}\tau_{\rm f}$ = Stokes number, proportional to radius of solid particles



(Johansen & Klahr 2005, Carballido et al. 2005)

Derivation of diffusion-sedimentation equilibrium

- The flux of dust particles in the vertical direction is

$$\mathcal{F}_z = \rho_{\rm p} v_z - D\rho_{\rm g}\frac{{\rm d}(\rho_{\rm p}/\rho_{\rm g})}{{\rm d}z}\,.$$

- Here we have assumed *Fickian diffusion* where the diffusive flux is proportional to the concentration $\epsilon = \rho_{\rm p}/\rho_{\rm g}$.
- In diffusion-sedimentation equilibrium we have $\mathcal{F} = 0$,

$$\epsilon v_z - D\frac{{\rm d}\epsilon}{{\rm d}z} = 0\,.$$

- We use the terminal velocity expression $v_z = -\tau_{\rm f}\Omega_{\rm K}^2 z$ to obtain

$$\frac{{\rm d}\ln\epsilon}{{\rm d}z} = -\frac{\tau_{\rm f}\Omega_{\rm K}^2 z}{D}$$

- The solution is

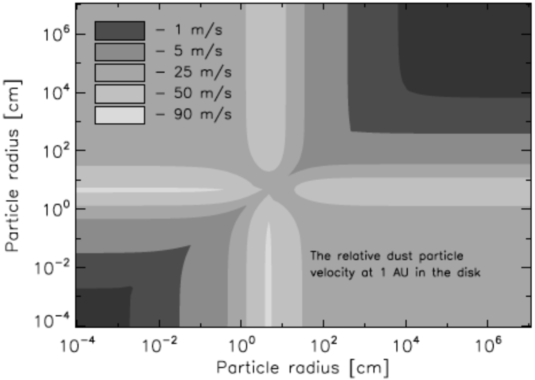
$$\epsilon(z) = \epsilon_{\rm mid}\exp[-z^2/(2H_\epsilon^2)]$$

- with

$$H_\epsilon^2 = \frac{D}{\tau_{\rm f}\Omega_{\rm K}^2} = \frac{\delta_{\rm t} H^2\Omega_{\rm K}}{\tau_{\rm f}\Omega_{\rm K}^2} \qquad \Rightarrow \qquad \frac{H_\epsilon^2}{H^2} = \frac{\delta_{\rm t}}{\Omega_{\rm K}\tau_{\rm f}}$$

Turbulent collision speeds

- Turbulent gas accelerates particles to high collision speeds:



(Brauer et al. 2008; based on Weidenschilling & Cuzzi 1993)

⇒ Small particles follow the same turbulent eddies and collide at low speeds

⇒ Larger particles collide at higher speeds because they have different trajectories

Terminal velocity approximation

- Equation of motion of particles ($\mathbf{v}$) and gas ($\mathbf{u}$)

$$
\begin{aligned}
\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} &= -\nabla\Phi - \frac{1}{\tau_\mathrm{f}}(\mathbf{v}-\mathbf{u}) \\
\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} &= -\nabla\Phi - \frac{1}{\rho}\nabla P
\end{aligned}
$$

- Particles do not care about the gas pressure gradient since they are very dense
- Subtract the two equations from each other and look for equilibrium

$$
\frac{\mathrm{d}(\mathbf{v}-\mathbf{u})}{\mathrm{d}t} = -\frac{1}{\tau_\mathrm{f}}(\mathbf{v}-\mathbf{u}) + \frac{1}{\rho}\nabla P = 0
$$

- In equilibrium between drag force and pressure gradient force the particles have their *terminal velocity* relative to the gas

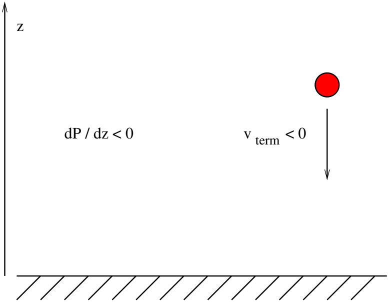
$$
\delta\mathbf{v} = \tau_\mathrm{f}\frac{1}{\rho}\nabla P
$$

$\Rightarrow$ **Particles move towards the direction of higher pressure**

Ball falling in Earth's atmosphere

$$\mathbf{v}_{\rm term} = \tau_{\rm f} \frac{1}{\rho} \boldsymbol{\nabla} P$$

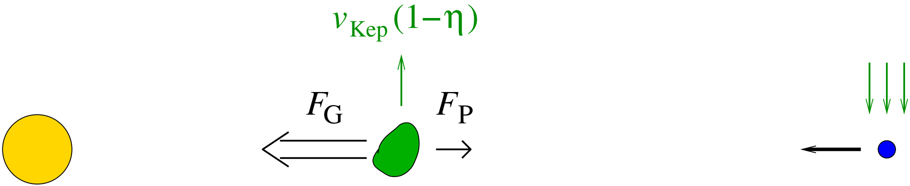
- Ball falling in Earth's atmosphere:



- Pressure is falling with height, so $\mathrm{d}P/\mathrm{d}z < 0$ and thus $v_{\rm term} < 0$

$\Rightarrow$ *Ball is seeking the point of highest pressure*

Radial drift



- Disc is hotter and denser close to the star
- Radial pressure gradient force mimics decreased gravity $\Rightarrow$ gas orbits slower than Keplerian
- Particles do not feel the pressure gradient force and want to orbit Keplerian
- Headwind from sub-Keplerian gas drains angular momentum from particles, so they spiral in through the disc
- Particles sublimate when reaching higher temperatures close to the star

Sub-Keplerian motion I

- Balance between gravity, centrifugal force and pressure gradient force:

$$0 = -\frac{GM_\star}{r^2} + \Omega^2 r - \frac{1}{\rho}\frac{\partial P}{\partial r}$$

- If we can ignore pressure gradients, then we recover the Keplerian solution

$$\Omega = \sqrt{\frac{GM_\star}{r^3}} \equiv \Omega_{\rm K}$$

- We can use $\Omega_{\rm K}$ to rewrite the original expression as

$$\Omega^2 r - \Omega_{\rm K}^2 r = \frac{1}{\rho}\frac{\partial P}{\partial r}$$

Sub-Keplerian motion II

- Balance between gravity, centrifugal force and pressure gradient force:

$$\Omega^2 r - \Omega_{\rm K}^2 r = \frac{1}{\rho}\frac{\partial P}{\partial r}$$

- Write pressure as $P = c_{\rm s}^2 \rho$

$$\Omega^2 r - \Omega_{\rm K}^2 r = \frac{c_{\rm s}^2}{P}\frac{r}{r}\frac{\partial P}{\partial r} = \frac{c_{\rm s}^2}{r}\frac{\partial \ln P}{\partial \ln r}$$

- Use $H = c_{\rm s}/\Omega_{\rm K}$ and get

$$\Omega^2 r - \Omega_{\rm K}^2 r = \frac{H^2 \Omega_{\rm K}^2}{r}\frac{\partial \ln P}{\partial \ln r}$$

- Divide equation by $\Omega_{\rm K}^2 r$

$$\left(\frac{\Omega}{\Omega_{\rm K}}\right)^2 - 1 = \frac{H^2}{r^2}\frac{\partial \ln P}{\partial \ln r}$$

Sub-Keplerian motion III

- Balance between gravity, centrifugal force and pressure gradient force:

$$\left(\frac{\Omega}{\Omega_\mathrm{K}}\right)^2 - 1 = \frac{H^2}{r^2}\frac{\partial \ln P}{\partial \ln r}$$

- The left-hand-side can be expanded as

$$\begin{aligned}
\left(\frac{\Omega}{\Omega_\mathrm{K}}\right)^2 - 1 &= \left(\frac{v}{v_\mathrm{K}}\right)^2 - 1 \\
&= \left(\frac{v_\mathrm{K} - \Delta v}{v_\mathrm{K}}\right)^2 - 1 \\
&= (1 - \Delta v/v_\mathrm{K})^2 - 1 \\
&= 1 - 2\Delta v/v_\mathrm{K} + (\Delta v/v_\mathrm{K})^2 - 1 \\
&\approx -2\Delta v/v_\mathrm{K} \quad \text{for} \quad \Delta v \ll v_\mathrm{K}
\end{aligned}$$

Sub-Keplerian motion IV

- Balance between gravity, centrifugal force and pressure gradient force:

$$-2\Delta v/v_{\rm K} = \left(\frac{H}{r}\right)^2 \frac{\partial \ln P}{\partial \ln r}$$

$$\Delta v = -\frac{1}{2}\left(\frac{H}{r}\right)^2 \frac{\partial \ln P}{\partial \ln r} v_{\rm K} \equiv -\eta v_{\rm K}$$

- Use $H/r = (c_{\rm s}/\Omega_{\rm K})/(v_{\rm K}/\Omega_{\rm K}) = c_{\rm s}/v_{\rm K}$ to obtain the final expression

$$\Delta v = -\frac{1}{2}\frac{H}{r}\frac{\partial \ln P}{\partial \ln r} c_{\rm s}$$

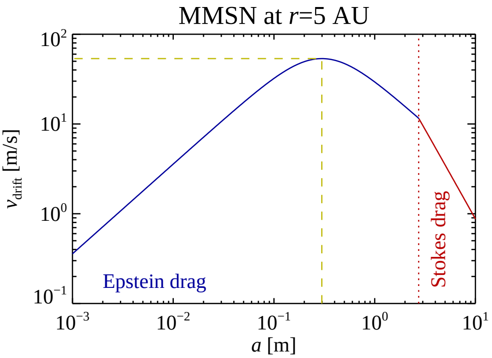
- Particles do not feel the global pressure gradient and want to orbit Keplerian $\Rightarrow$ headwind from the sub-Keplerian gas

Radial drift

Balance between drag force and head wind gives radial drift speed
(Adachi et al. 1976; Weidenschilling 1977)

$$v_{\rm drift} = -\frac{\Delta v}{\Omega_{\rm K}\tau_{\rm f} + (\Omega_{\rm K}\tau_{\rm f})^{-1}}$$

for Epstein drag law $\tau_{\rm f} = R\rho_\bullet/(c_{\rm s}\rho_{\rm g})$

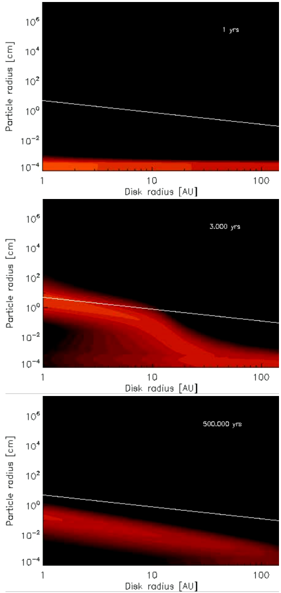


- MMSN $\Delta v \sim 50\ldots100$ m/s
  (Cuzzi et al. 1993; Chiang & Goldreich 1997)
- Drift time-scale of 100 years for particles of 30 cm in radius at 5 AU

Coagulation and radial drift

- Coagulation equation of dust particles can be solved by numerical integration
- Brauer et al. (2008) started with μm-sized particles and let the size distribution evolve by sticking and fragmentation
- The head wind from the gas causes cm particles to spiral in towards the star

$\Rightarrow$ All solid material lost to the star within a few million years



(Brauer et al. 2008)

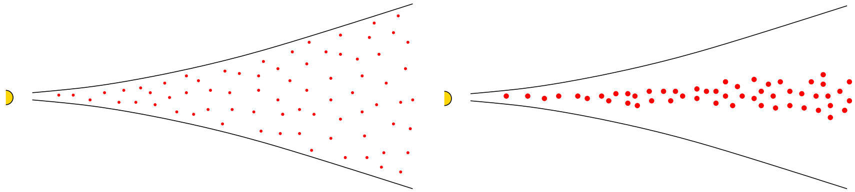
Meter barrier

1. Large particles do not stick well

2. Particles bounce or shatter each other when collision speeds are higher than $\sim$1 m/s

3. Pebbles, rocks, and boulders drift rapidly through the disc because of the headwind of the sub-Keplerian gas

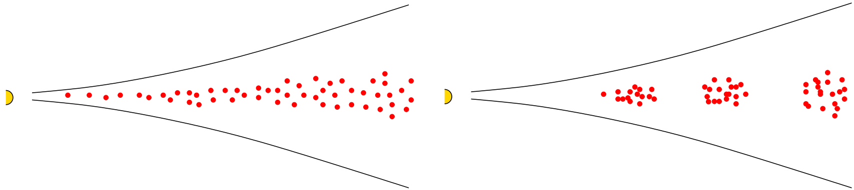
Gravitational instability



- Dust and ice particles in a protoplanetary disc coagulate to cm-sized pebbles and rocks
- Pebbles and rocks *sediment* to the mid-plane of the disc
- Further growth frustrated by high-speed collisions ($>$1–10 m/s) which lead to erosion and bouncing *(Blum & Wurm 2008)*
- Layer *not* dense enough for gravitational instability *(Goldreich & Ward 1973; Weidenschilling & Cuzzi 1993)*

$\Rightarrow$ **Need some way for particle layer to get dense enough to initiate gravitational collapse**

How turbulence aids planetesimal formation



1. *Passive concentration* as particles pile up in long-lived pressure bumps and vortices excited in the turbulent gas flow
   (Barge & Sommeria 1995; Klahr & Bodenheimer 2003; Johansen et al. 2009a)

2. *Active concentration* as particles make dense filaments and clumps to protect themselves from gas friction
   (Youdin & Goodman 2005; Johansen & Youdin 2007; Johansen et al. 2009b; Bai & Stone 2010a,b,c)

Pressure bumps

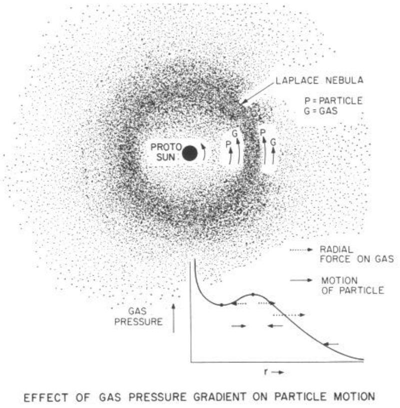


Fig. 1.

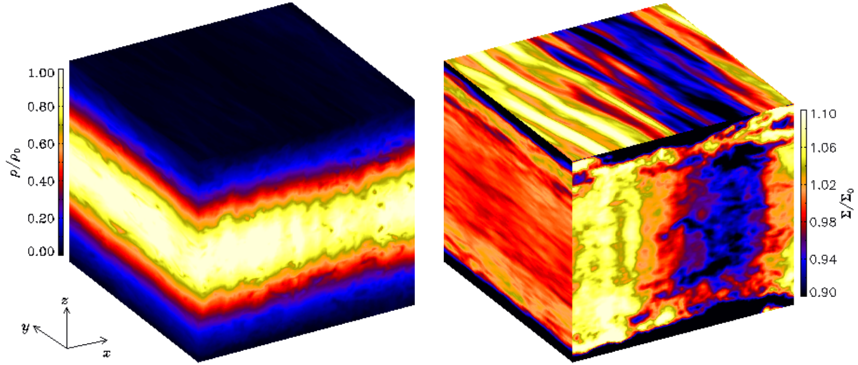
(Figure from Whipple 1972)

- Particles seek the point of highest pressure

⇒ Particles get trapped in *pressure bumps*

- Achieve high enough *local* density for gravitational instability and planetesimal formation

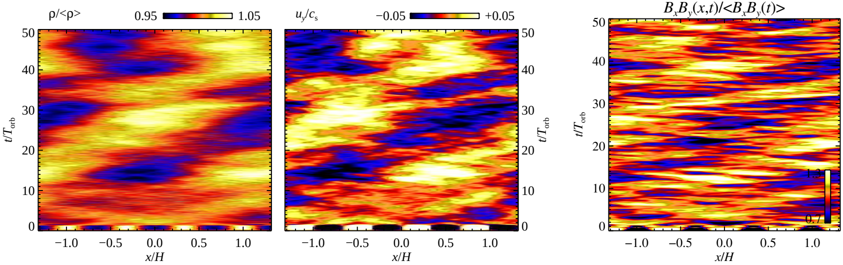
High-pressure regions



(Johansen, Youdin, & Klahr 2009)

- Gas density shows the expected vertical stratification
- Gas column density shows presence of large-scale pressure fluctuations with variation only in the radial direction
- Pressure fluctuations of order 10%

Stress variation and pressure bumps



- Mass accretion rate and column density:

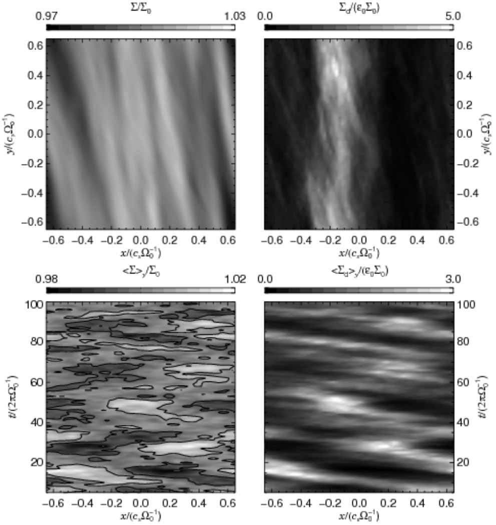
$$\dot{M} = 3\pi\Sigma\nu_{\rm t} \quad \Rightarrow \quad \Sigma = \frac{\dot{M}}{3\pi\nu_{\rm t}}$$

$$\nu_{\rm t} = \alpha c_{\rm s} H$$

$\Rightarrow$ Constant $\dot{M}$ and constant α yield $\Sigma \propto r^{-1}$

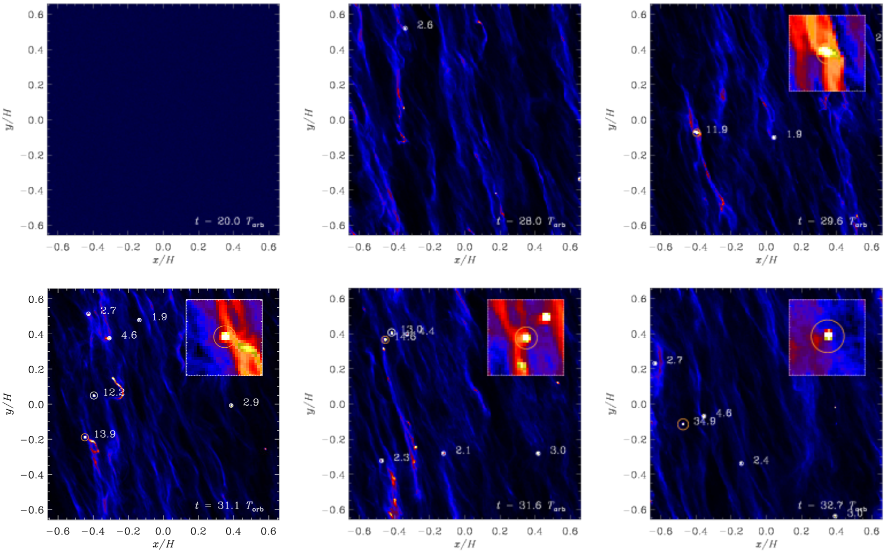
$\Rightarrow$ Radial variation in α gives pressure bumps

Particle trapping



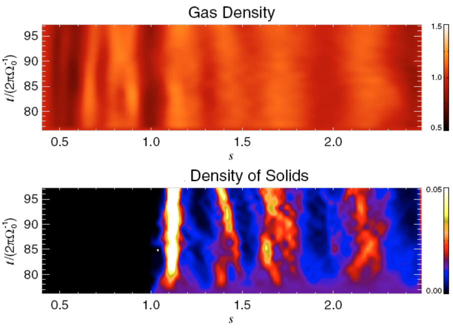
- Strong correlation between high gas density and high particle density
  (Johansen, Klahr, & Henning 2006)

Forming planetesimals in pressure bumps



(Johansen et al. 2011)

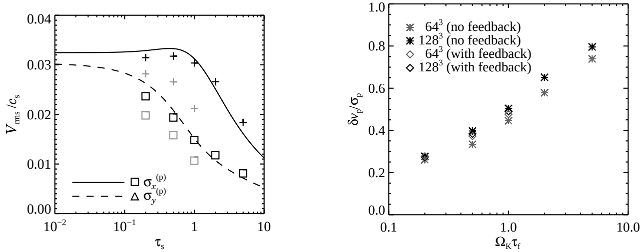
What sets the scale of pressure bumps?



- Pressure bumps reported in a number of MRI papers *(Fromang & Stone 2009; Davis et al. 2010; Simon et al. 2012)*
- Pressure bumps cascade to the largest scales of local box simulations, but may stop at 5–10 scale heights *(Johansen et al. 2009; Dittrich, Klahr, & Johansen 2013)*
- More global simulations needed! *(e.g. Lyra et al. 2008; Uribe et al. 2012)*

The double-edged sword called turbulence

☺ Turbulence can excite long-lived pressure bumps which trap particles
☹ Turbulence excites high relative particle speeds

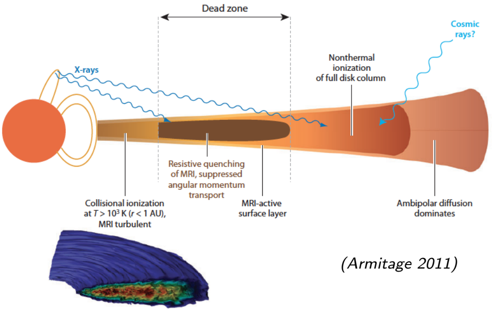


(Youdin & Lithwick 2008; Johansen et al. 2007)

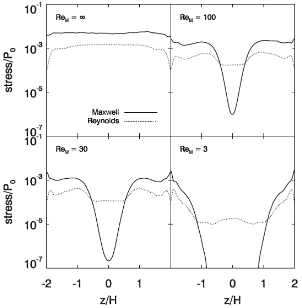
- Gas has rms speed $v \sim \sqrt{\alpha} c_s$
- The particle rms speed falls with increasing size
- The particle collision speed approaches the particle rms speed with increasing size

⇒ Boulders have $v_{coll} \sim 50$ m/s for $\alpha = 0.01$

Dead zone and layered accretion



(Armitage 2011)



(Gammie 1996; Fleming & Stone 2003; Oishi et al. 2007)

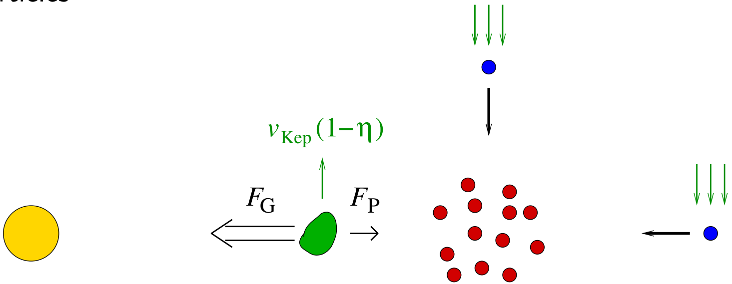
- Cosmic rays do not penetrate to the mid-plane of the disc, so the ionisation fraction in the mid-plane is too low to sustain MRI

$\Rightarrow$ Accretion in active surface layers or by disc winds
(Blandford & Payne 1982; Fromang et al. 2012; Bai & Stone 2013)

$\Rightarrow$ Weak turbulence and low collision speeds in the *dead zone*

Streaming instability

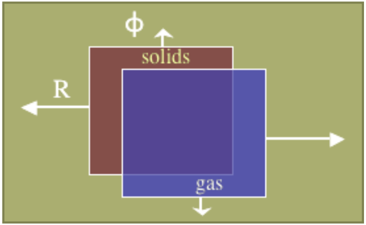
- Gas orbits slightly slower than Keplerian
- Particles lose angular momentum due to headwind
- Particle clumps locally reduce headwind and are fed by isolated particles

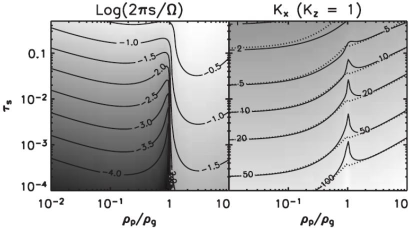


$\Rightarrow$ Youdin & Goodman (2005): **"Streaming instability"**

- *Shear instabilities* such as Kelvin-Helmholtz instability and magnetorotational instability feed on spatial variation in the gas velocity
- *Streaming instabilities* feed on velocity difference between two components (gas and particles) at the same location

Linear analysis

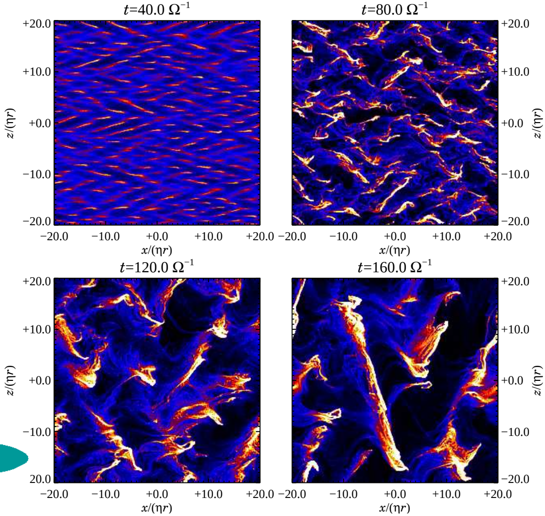




- The streaming feeds off the velocity difference between gas and particles
- Particles move faster than the gas and drift inwards, pushing the gas outwards
- In total there are 8 linear modes (density waves modified by drag)
- *One* of the modes is unstable *(Youdin & Goodman 2005; Jacquet, Balbus, & Latter 2011)*
- Requires both radial and vertical displacements
- Fastest growth for large particles and local dust-to-gas ratio above unity

Clumping

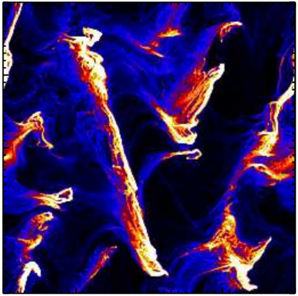
Linear and non-linear evolution of radial drift flow of meter-sized boulders:



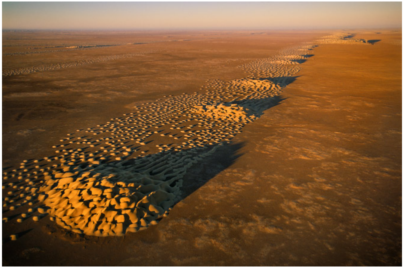
⇒ Strong clumping in non-linear state of the streaming instability

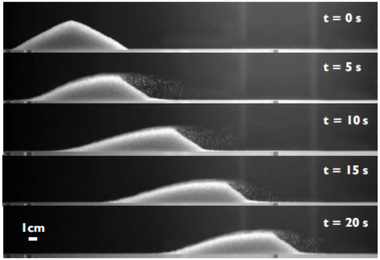
(Youdin & Johansen 2007; Johansen & Youdin 2007; done with Pencil Code [pencil-code.googlecode.com]*)*

Why clump?



Sand dunes



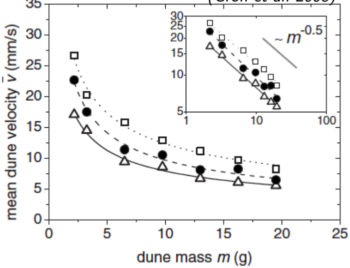


- *Barchan* sand dunes form when sparse sand moves over bedrock and wind has a dominant direction
- Experiments show that larger sand dunes move slower than smaller sand dunes

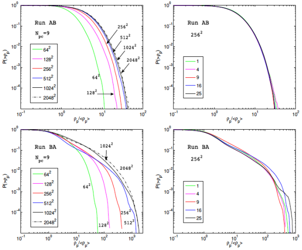
⇒ Small sand dunes melt together to larger and larger sand dunes

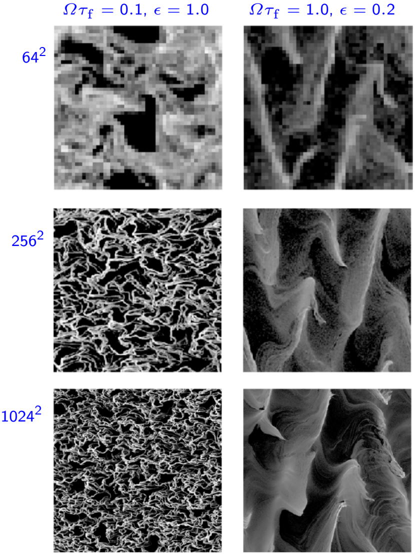
- Similar dynamics to what drives formation of dense filaments of particles in protoplanetary discs...

(Groh et al. 2008)



Convergence tests – unstratified





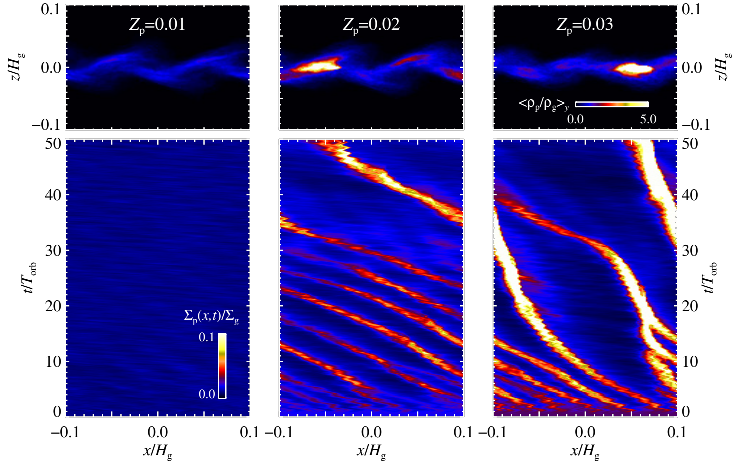
- Bai & Stone (2010a) presented high-resolution convergence tests of non-stratified 2-D simulations

$\Rightarrow$ Maximum particle density increases with resolution, converging at 1024^2 or 2048^2.

$\Rightarrow$ Confirmation of Pencil Code results with independent code (Athena)

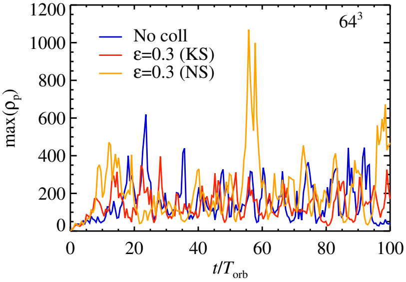
Stratified simulations

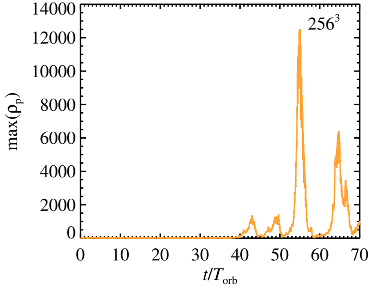
- Johansen, Youdin, & Mac Low (2009b) presented first stratified simulations of streaming instabilities
- Particles sizes $\Omega\tau_{\rm f} = 0.1, 0.2, 0.3, 0.4$ (3–12 cm at 5 AU, 1–4 cm at 10 AU)
- Dust-to-gas ratio no longer a free parameter, but column density $Z = \Sigma_{\rm p}/\Sigma_{\rm g}$ is



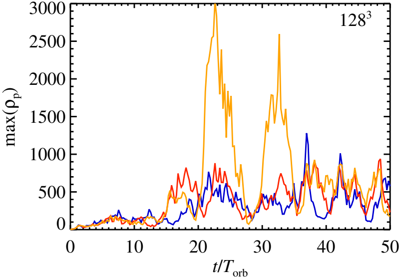
Convergence tests – stratified

- Particle density up to 3000 times local gas density
- Criterion for gravitational collapse: $\rho_\mathrm{p} \gtrsim \Omega^2/G \sim 100\rho_\mathrm{g}$
- Maximum density increases with increasing resolution



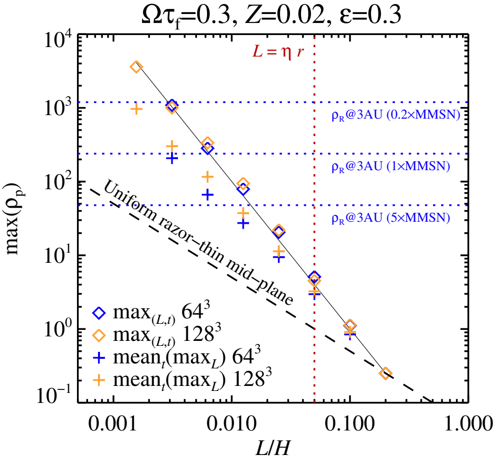


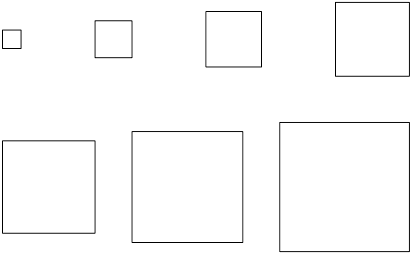
(Johansen, Mac Low, & Lacerda, in preparation)



(Johansen, Youdin, & Lithwick 2012)

Scale-by-scale convergence





- Plot shows maximum density over a given scale (averaged over time)
- Points for 64^3 and 128^3 almost on top of each other

$\Rightarrow$ Streaming instability overdensities converge scale-by-scale

- Increasing the resolution increases the maximum density because density at grid-cell level gains structure at increased resolution

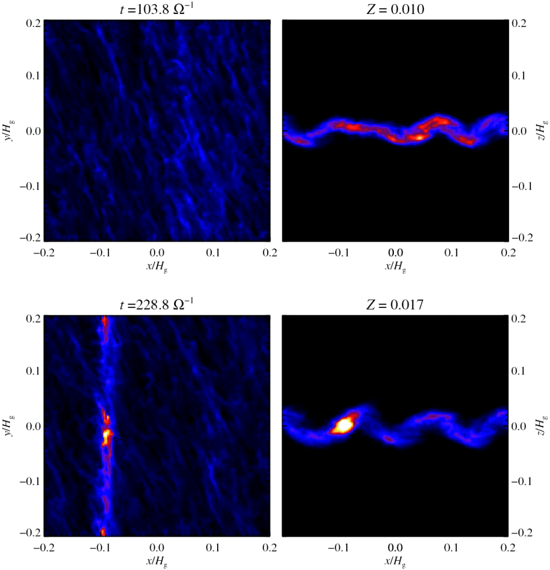
Sedimentation of 10 cm rocks

- Streaming instability relies on the ability of solid particles to accelerate the gas towards the Keplerian speed

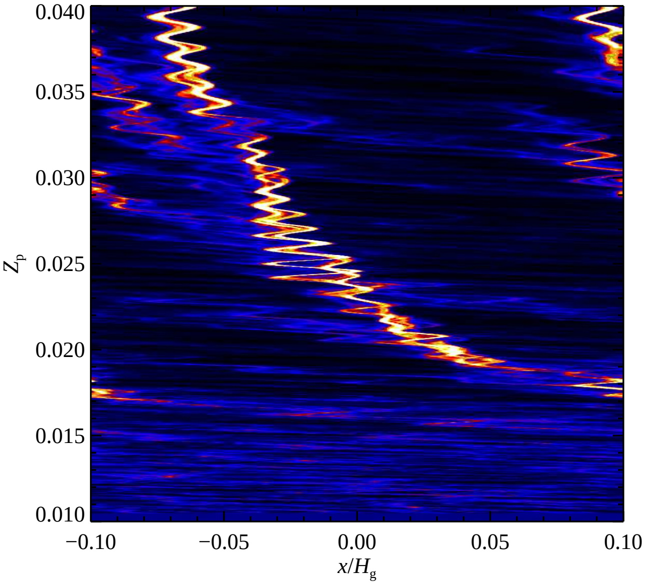
⇒ Efficiency increases with the metallicity of the gas

- Solar metallicity: turbulence caused by the streaming instability puffs up the mid-plane layer, but no clumping

- Dense filaments form spontaneously above $Z \approx 0.015$

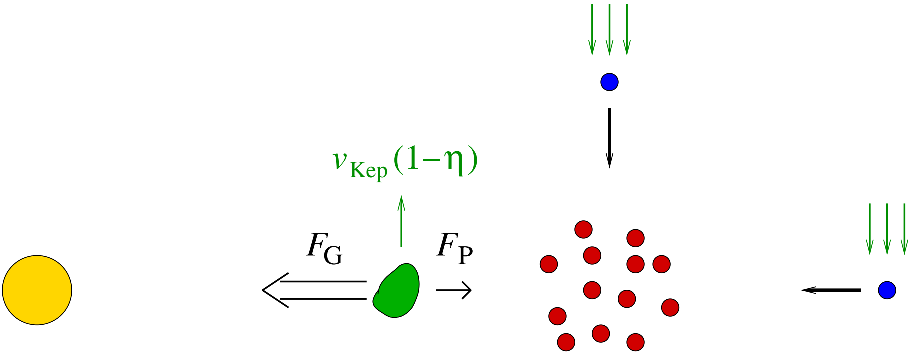


Metallicity matters



Why is metallicity important?

- Gas orbits slightly slower than Keplerian
- Particles lose angular momentum due to headwind
- Particle clumps locally reduce headwind and are fed by isolated particles



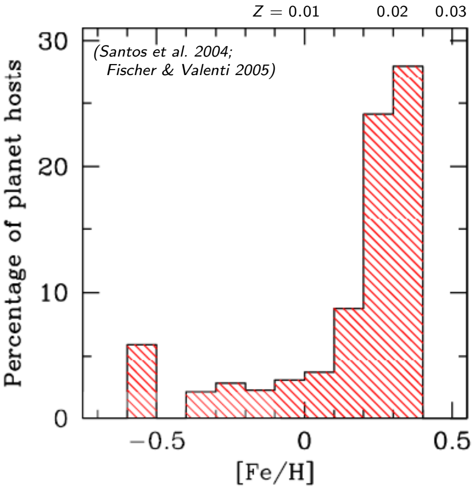
- *Clumping relies on particles being able to accelerate the gas towards Keplerian speed*

Metallicity of exoplanet host stars

- First planet around solar-type star discovered in 1995
  (Mayor & Queloz 1995)

- Today more than 800 exoplanets known

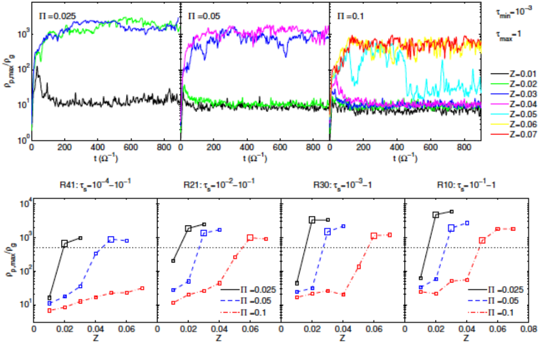
- Exoplanet probability increases sharply with metallicity of host star



⇒ Expected due to efficiency of core accretion
(Ida & Lin 2004; Mordasini et al. 2009)

⇒ . . . but planetesimal formation may play equally big part
(Johansen et al. 2009; Bai & Stone 2010b)

Dependence on headwind parameter



- Bai & Stone (2010c) searched for the critical metallicity for clumping as a function of the headwind parameter $\Pi = \Delta v / c_\mathrm{s}$

$\Rightarrow$ Slow headwind (close to star or in pressure bumps) gives lower threshold

$\Rightarrow$ Careful when using pressure bumps to stop radial drift – streaming instability leads to strong clumping when headwind is slow

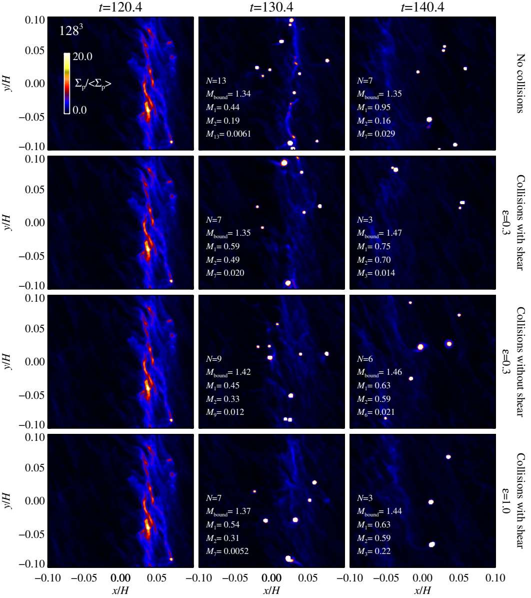
Planetesimal masses

- Largest planetesimals are typically 500 km in radius
- Comparable to dwarf planet Ceres

⇒ Asteroids born big?
(Morbidelli et al. 2009)

- A number of smaller 100-km scale planetesimals form alongside the large ones
- Scaling to Kuiper belt gives twice as large planetesimals

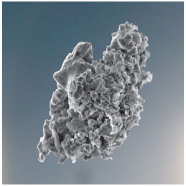
⇒ May explain why Kuiper belt objects are larger than asteroids



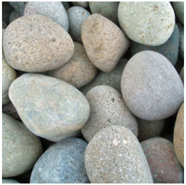
(Johansen, Youdin, & Lithwick 2012)

The "clumping scenario" for planetesimal formation

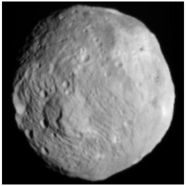
1. Dust growth by coagulation to mm-cm-dm-sized pebbles



2. Spontaneous clumping through streaming instabilities and/or in pressure bumps and vortices



3. Gravitational collapse to form 100–1000 km radius planetesimals



Three ways to clump

Three ways to concentrate in protoplanetary discs:

1. Concentration between turbulent eddies at the Kolmogorov scale ($\lambda \sim 1$ km, eddy turn-over time $\tau_\eta \sim 1000$ s)
   (Cuzzi et al. 2008; Padoan et al. 2006; Pan et al. 2011)
2. Streaming instabilities ($\lambda \sim 0.1 H \sim 10^6$-$10^7$ km)
   (Youdin & Goodman 2005; Johansen & Youdin 2007; Johansen et al. 2009b, Bai & Stone 2010)
3. Concentration in pressure bumps ($\lambda \sim 5 H \sim 10^8$ km)
   (Whipple 1972; Haghighipour & Boss 2003; Johansen et al. 2006; Johansen et al. 2009a)

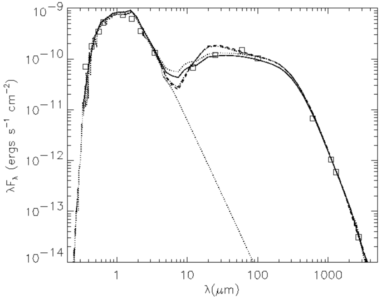
The optimally trapped particle has friction time

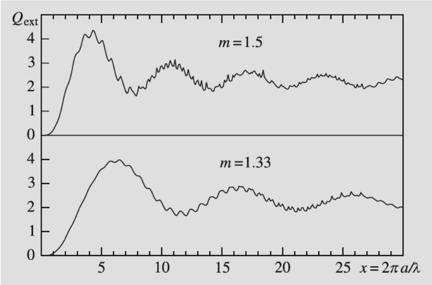
1. $\tau_\mathrm{f} \sim \tau_\eta$ ($R \sim 1$ mm, depends on turbulence strength)
2. $\tau_\mathrm{f} \sim 1/\Omega_\mathrm{K}$ ($R \sim 10$ cm)
3. $\tau_\mathrm{f} \sim 1/\Omega_\mathrm{K}$ ($R \sim 10$ cm)

Some open questions in planetesimal formation

1. Formation of pebbles
2. Do chondrules stream?
3. Pebble accretion
4. Asteroids and Kuiper belt objects

Observed dust growth in protoplanetary discs

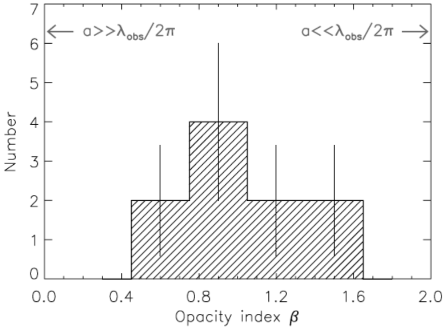




- Dust opacity as a function of frequency $\nu = c/\lambda$:
  - $\kappa_\nu \propto \nu^2$ for $\lambda \gg a$
  - $\kappa_\nu \propto \nu^0$ for $\lambda \ll a$
- $F_\nu \propto \nu^\alpha \propto \kappa_\nu B_\nu \propto \kappa_\nu \nu^2 \propto \nu^\beta \nu^2$
- By measuring α from SED, one can determine β from $\beta = \alpha - 2$
- Knowledge of β gives knowledge of dust size

Opacity index

- Rodmann et al. 2006 observed 10 low-mass pre-main-sequence stars in the Taurus-Auriga star-forming region
- All had $\beta \sim 1$, indicating growth to at least millimeters

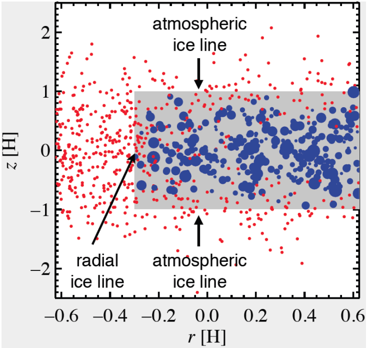


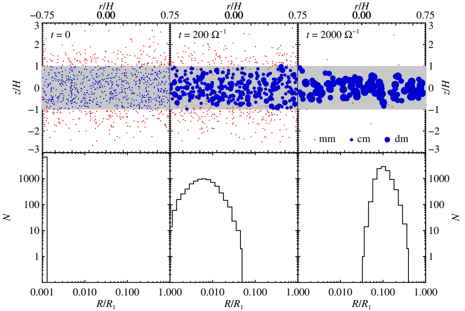
- The disc around TW Hya contains $10^{-3}\ M_{\odot}$ of cm-sized pebbles (Wilner et al. 2005)

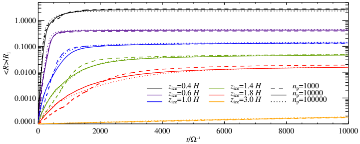
Formation of icy pebbles

- Pebbles are observed in abundance in nearby protoplanetary discs
  (e.g. Testi et al. 2003; Wilner et al. 2005)
- How do pebbles form so efficiently?
- *Near ice lines pebbles can form like hail stones*

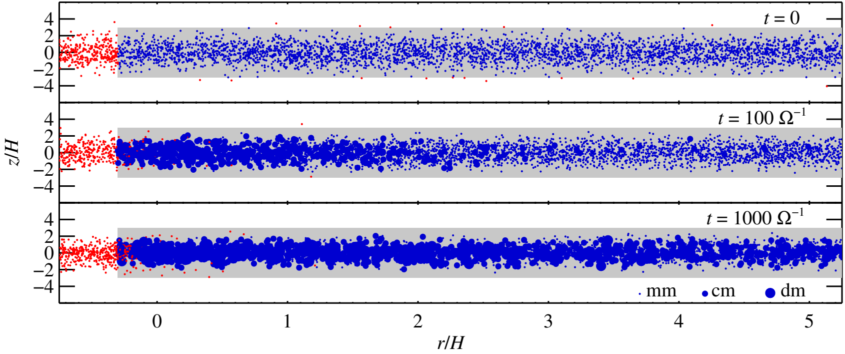
⇒ Efficient formation of cm-dm sized pebbles near the water ice line at 3 AU *(Ros & Johansen, submitted)*







Radial iceline



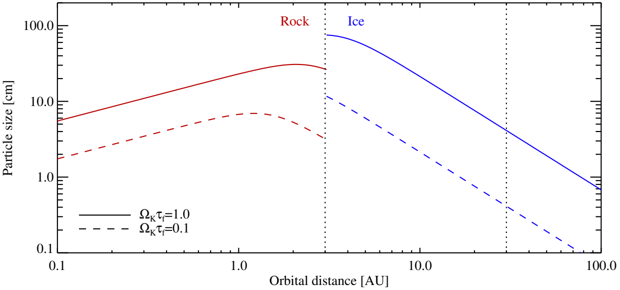
- The radial ice-line feeds vapour directly into the mid-plane

⇒ Growth to dm-sized ice balls

⇒ Turbulent diffusion mixes growing pebbles in the entire cold region

⇒ Future models of coagulation *and* condensation could yield large enough particle sizes for streaming instabilities to become important

Do chondrules stream?

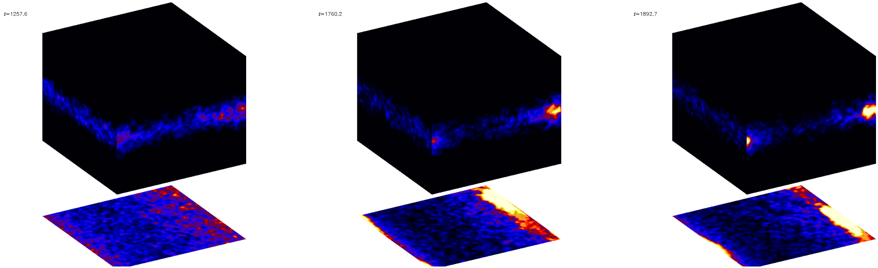


- Streaming instability most efficient for particles with $\tau_{\rm f} \sim (0.1 \ldots 1)\Omega_{\rm K}^{-1}$
- At 2–3 AU particles with $\Omega\tau_{\rm f} = 0.1$ are 2-4 cm in size

$\Rightarrow$ Do chondrite parent bodies form at a time when 90% of the nebula gas was gone? [minimum particle size 2-4 mm instead]

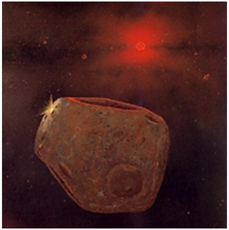
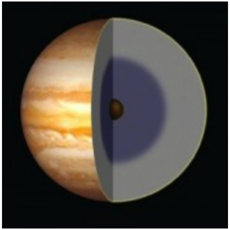
$\Rightarrow$ Are chondrules splash ejecta from a first generation of planetesimals? *(Sanders & Taylor 2005; Asphaug et al. 2011)*

Streaming instability in small particles



- Hard to obtain clumping by streaming instabilities for small particles
- Preliminary results from Daniel Carrera's PhD thesis in Lund show a metallicity threshold for mm-sized particles of $Z = 0.035$
- Need photoevaporation of gas or radial drift and pile-up of refractory particles in the terrestrial planet formation region

Classical core accretion scenario

1. Dust grains and ice particles collide to form km-scale planetesimals
2. Large protoplanet grows by run-away accretion of planetesimals
3. Protoplanet attracts hydrostatic gas envelope
4. Run-away gas accretion as $M_{\rm env} \approx M_{\rm core}$
5. Form gas giant with $M_{\rm core} \approx 10 M_\oplus$ and $M_{\rm atm} \sim M_{\rm Jup}$

(e.g. Safronov 1969, Mizuno 1980, Pollack et al. 1996, Rafikov 2004)

Core formation time-scales

- The size of the protoplanet relative to the Hill sphere:

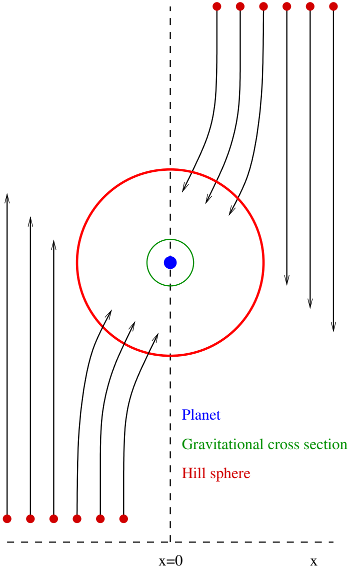
$$\frac{R_\mathrm{p}}{R_\mathrm{H}} \equiv \alpha \approx 0.001 \left(\frac{r}{5\,\mathrm{AU}}\right)^{-1}$$

- Maximal growth rate

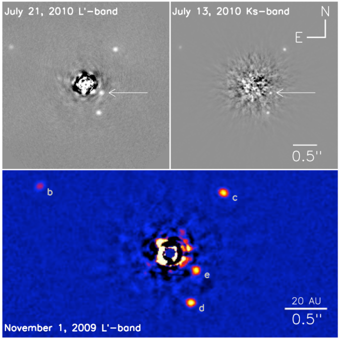
$$\dot{M} = \alpha R_\mathrm{H}^2 \mathcal{F}_\mathrm{H}$$

$\Rightarrow$ Only 0.1% (0.01%) of planet- esimals entering the Hill sphere are accreted at 5 AU (50 AU)

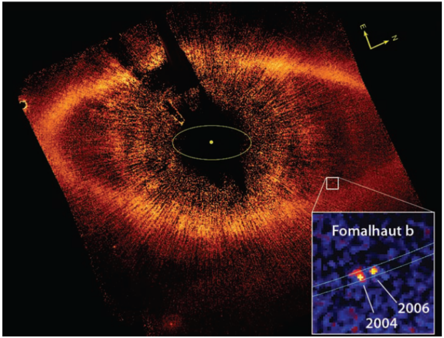
$\Rightarrow$ Time to grow to 10 $M_\oplus$ is
$\sim$10 Myr at 5 AU
$\sim$50 Myr at 10 AU
$\sim$5,000 Myr at 50 AU



Directly imaged exoplanets



(Marois et al. 2008; 2010)

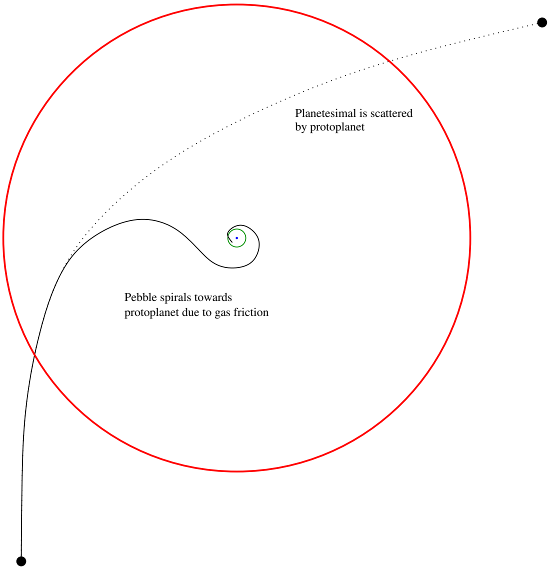


(Kalas et al. 2008)

- HR 8799 (4 planets at 14.5, 24, 38, 68 AU)
- Fomalhaut (1 controversial planet at 113 AU)

⇒ No way to form the cores of these planets within the life-time of the protoplanetary gas disc *by standard core accretion*

Pebble accretion



- Most planetesimals are simply scattered by the protoplanet
- Pebbles spiral in towards the protoplanet due to gas friction

⇒ Pebbles are accreted from the entire Hill sphere

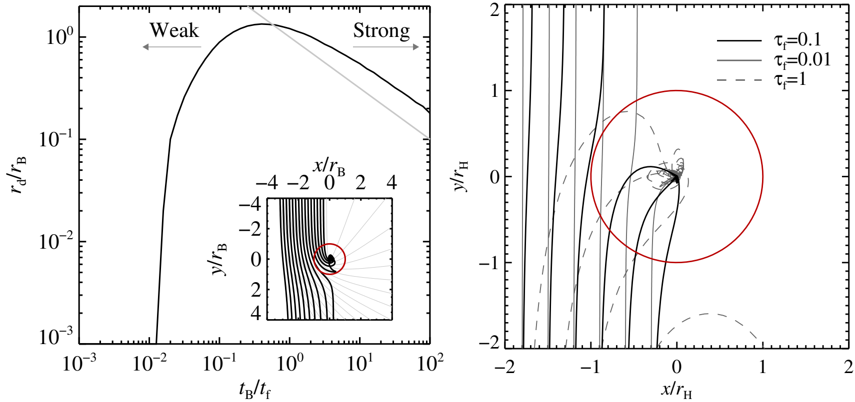
- Growth rate by planetesimal accretion is

$$\dot{M} = \alpha R_{\mathrm{H}}^2 \mathcal{F}_{\mathrm{H}}$$

- Growth rate by pebble accretion is

$$\dot{M} = R_{\mathrm{H}}^2 \mathcal{F}_{\mathrm{H}}$$

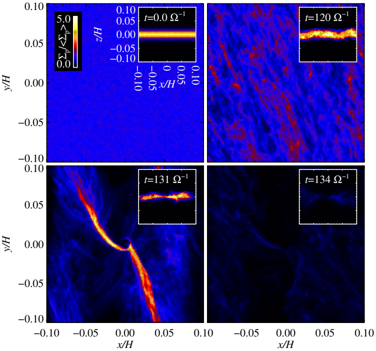
Pebble accretion regimes

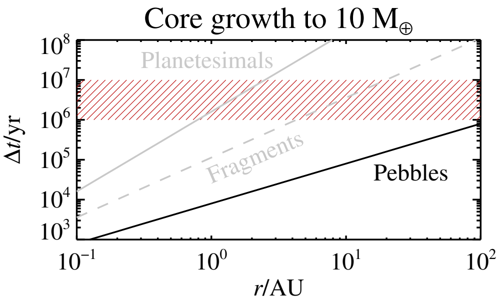


Two main pebble accretion regimes:

1. Bondi regime (when $\Delta v \gg v_{\rm H}$)
   Particles pass the core with speed Δv, giving $\dot{M} \propto r_{\rm B}^2 \propto M^2$
2. Hill regime (when $\Delta v \ll v_{\rm H}$)
   Particles enter Hill sphere with speed $v_{\rm H} \approx \Omega R_{\rm H}$, giving $\dot{M} \propto M^{2/3}$

Time-scale of pebble accretion





⇒ Pebble accretion speeds up core formation by a factor 1,000 at 5 AU and a factor 10,000 at 50 AU

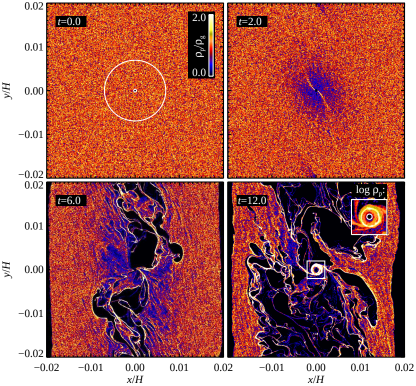
(Lambrechts & Johansen 2012; Nesvorny & Morbidelli 2012; see also Ormel & Klahr 2010)

⇒ Cores form well within the life-time of the protoplanetary gas disc, even at large orbital distances

- Requires large planetesimal seeds to accrete in Hill regime, consistent with turbulence-aided planetesimal formation

Growth of planetesimals

Put large (500 km) planetesimal in an ocean of pebbles:

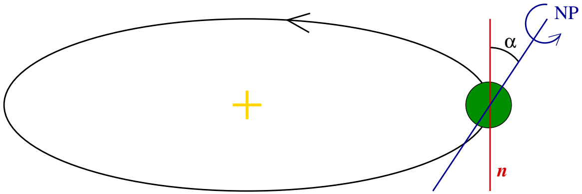


⇒ *Prograde* accretion disc forms around the protoplanet
(Johansen & Lacerda 2010)

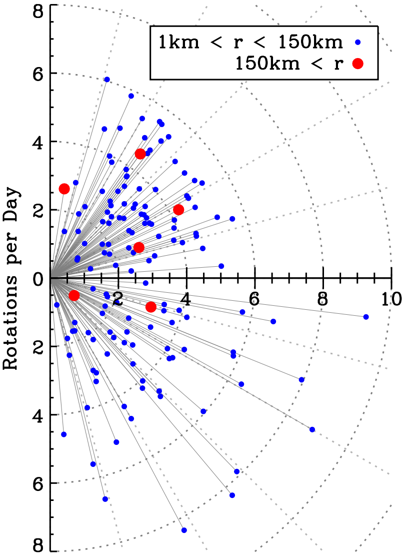
Asteroid rotation

- The majority of large asteroids have axial tilt $\alpha < 90^\circ$
- Called "direct" or "prograde" rotation

Body	α
Ceres	2^o
2 Pallas	60^o
3 Juno	50^o
4 Vesta	29^o
5 Astraea	33^o
6 Hebe	42^o

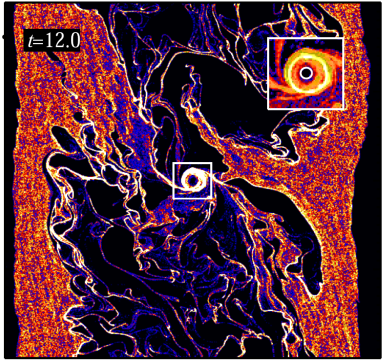


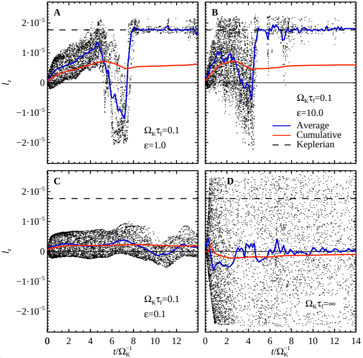
Asteroid poles



- Plot of asteroid pole axes
  (from Johansen & Lacerda 2010)
- Largest asteroids have a *tendency* to rotate prograde (1-2 σ)
- ... but there is a very large scatter
- The two large retrograde asteroids, 2 Pallas and 10 Hygiea, are actually spinning on the side
- Spin is normally attributed to the random effect of large impacts
  (Dones & Tremaine 1993)

Spin of asteroids and Kuiper belt objects

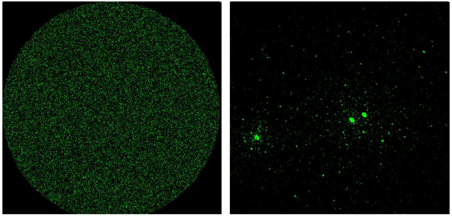




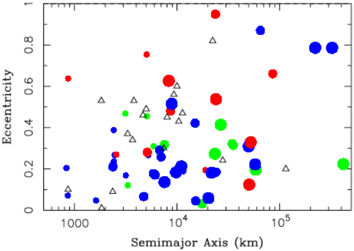
$\Rightarrow$ Prograde rotation with $P \approx (5 \ldots 10)\,\mathrm{h}$ induced for particles between centimeters and a few meters

$\Rightarrow$ Predict that pristine Kuiper belt objects also have prograde spin

Kuiper belt binaries

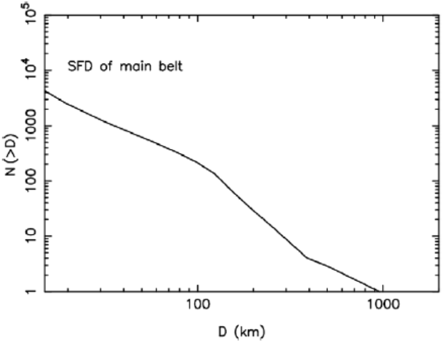


- At least 30% of 100-km classical KBOs are binaries
- Nesvorny et al. (2010) modelled gravitational collapse of particle clumps to explain why binary KBO can have similar colors
- Found good statistical agreement in orbital parameters between simulations and observed KBO binary systems
- Almost no binaries in scattered disc – ionised by close encounters?
- Need better models of collapsing pebble clouds (e.g. Tristen Hayfield's work in Heidelberg and Kalle Jansson's PhD thesis in Lund)



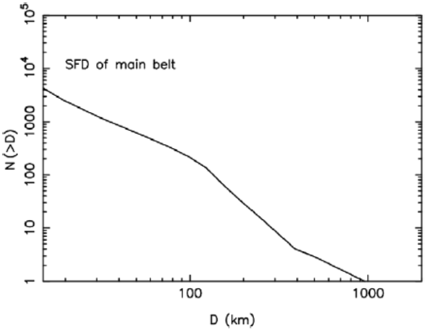
Were asteroids born BIG?

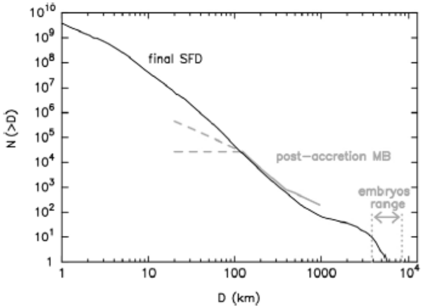
Morbidelli et al. (2009) performed simulations to reconstruct the current observed size distribution of asteroids:



- Plot shows the *cumulative size distribution*, i.e. number of asteroids with a size larger than a given value
- Asteroid belt mainly depleted by resonances with Jupiter
- Depletion is size-independent
- Assume that reconstructed post-accretion asteroid belt had 100 times more mass in all bins than today's asteroid belt

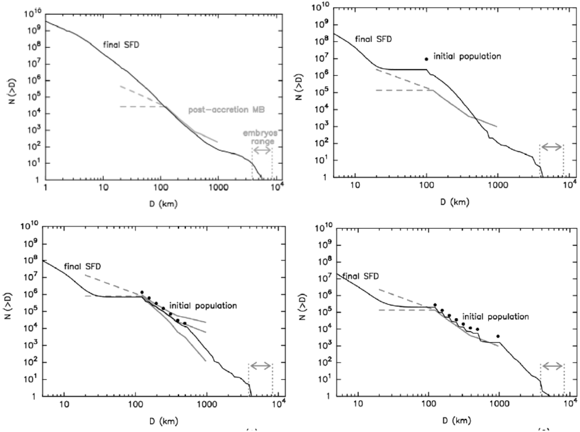
Starting with km-sized planetesimals





⇒ **Problem**: accumulation of asteroids from km-sized planetesimals does not reproduce the observed bump at 100 km

Various assumptions



- Start with different planetesimal sizes (1 km, 100 km, 100–500 km, 100-1000 km)

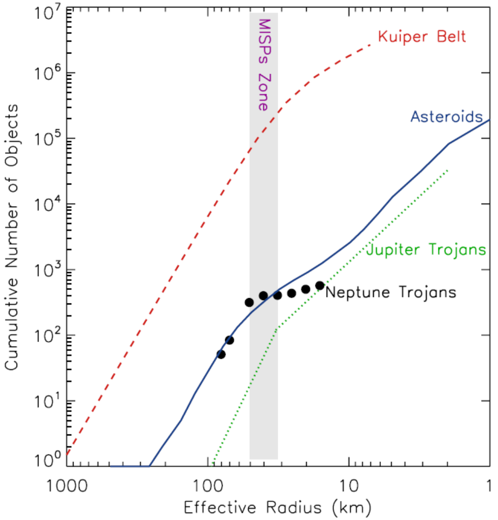
⇒ Need to start with BIG planetesimals to explain both the lack of <100 km asteroids and the slope of the 100-1000 km asteroids

! Weidenschilling (2011) instead proposed that asteroids were born small

? What is the role of pebble accretion in sculpting the asteroid belt?

Missing intermediate-size planetesimals

- Sheppard & Trujillo (2010) searched for Neptune Trojans
- Sensitive to planetesimals larger than 16 km
- Found no Trojans with radius less than 45 kilometers
- Dubbed them *the missing intermediate-size planetesimals* (MISPs)



Future

- Models and experiments of dust coagulation / fragmentation / bouncing are very advanced now
- Ice condensation may be another necessary ingredient for efficient formation of pebbles
- Particle clumping by streaming instabilities / pressure bumps / vortices is by now a robust phenomenon studied by several groups with independent codes
- Pebble accretion is very efficient at growth from planetesimals to planets – the full importance of this new growth mechanism is still being explored
- Asteroid belt and Kuiper belt may be sculpted by gravitational collapse, pebble accretion *and* planetesimal collisions

FUTURE : COMBINE IT ALL

Discussion

Suggestions by Alessandro Morbidelli:

- How can we understand that planetesimal formation was protracted for $\sim$4 Myr (i.e. from iron meteorite parent bodies, formed at CAI time, to chondritic parent bodies formed about 4 Myr later)?
- Do planetesimals that form at significantly different times (e.g. differentiated and undifferentiated ones) form at different places or can they represent different accretion episodes in the same place of the disc?
- Could collisional erosion of large planetesimals and accretion of new planetesimals from pebbles coexist in the same place of the disc?
- Why is the basic constituents of chondritic meteorites sub-mm objects (CAIs, chondrules) whereas streaming instabilities prefer cm-sized pebbles?
- How much mixing do we expect in planetesimal formation models? How do we understand the diversity of meteorite classes?