Leibniz-Institut für Astrophysik Potsdam
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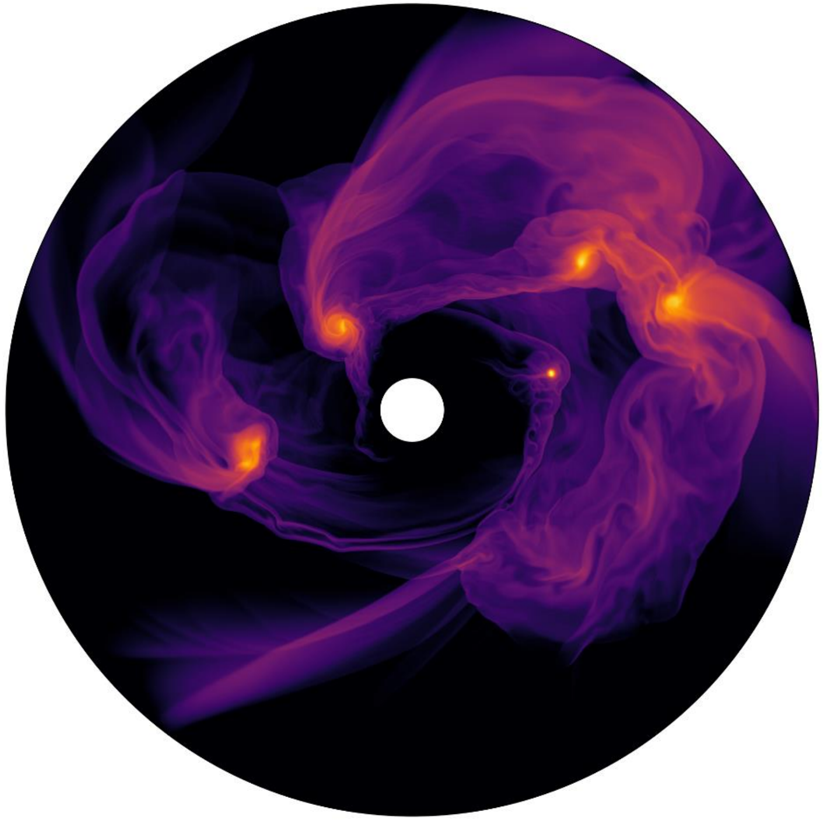
ERC, Epoch-of-Taurus, 101043302

Self-gravity processes in protoplanetary disks



Steven RENDON RESTREPO
Magnetohydrodynamics and Turbulence
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Grenoble, Nice, Toulouse, 2024

ERC Consolidator Grant - Early phases of planetary birth sites: environmental context and interstellar inheritance

ERC, Epoch-of-Taurus, 101043302

MHD and turbulence

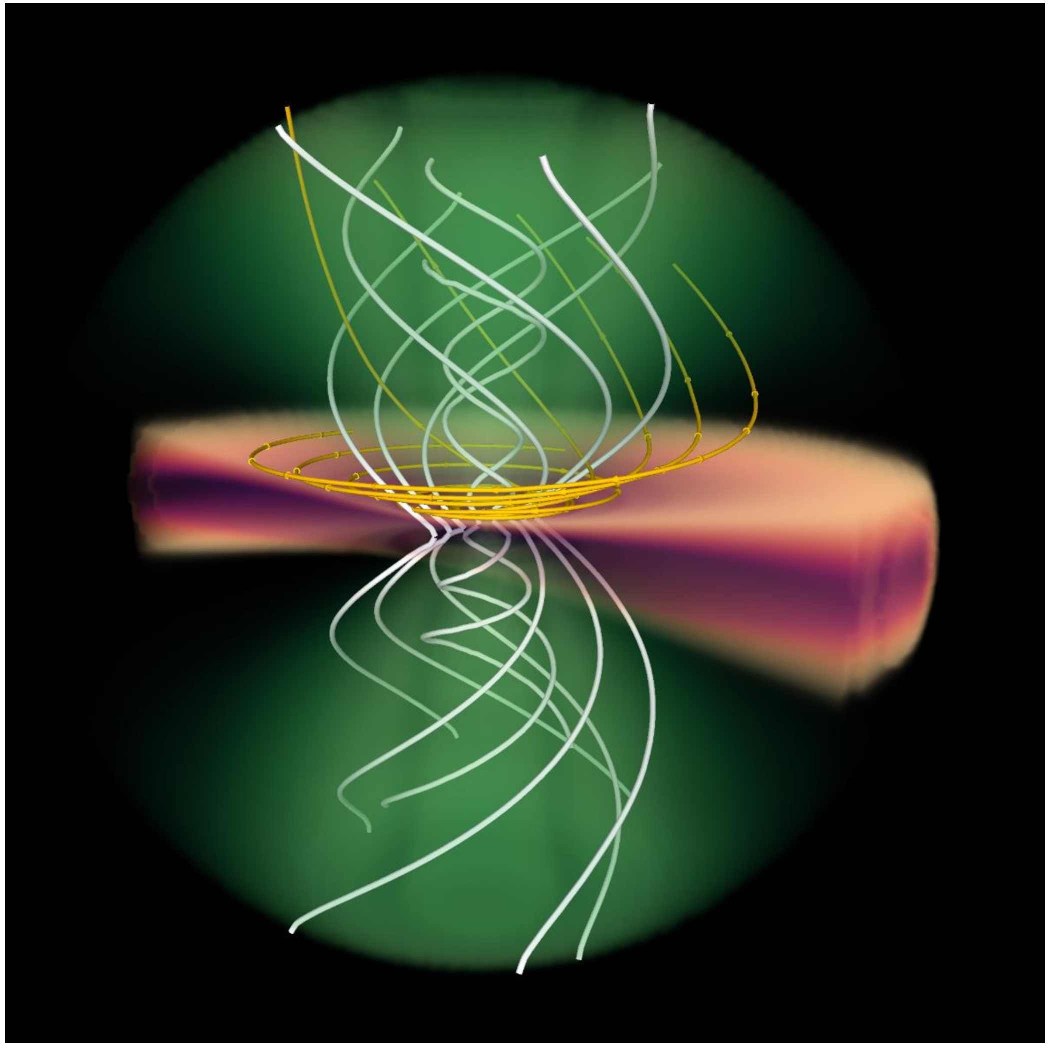
Head: Dr. Oliver Gressel

Post-Docs:
Steven Rendon Restrepo
Marc Van den Bossche (joining in Oct 2024)
Ankush Mandal (joining in Oct 2024)
Eleftheria Sarafidou (DFG grant)

PhD: Andrej HERMANN, Jibin JOSEPH, Noa Hoffmann (joining in Oct 2024)

PhD with DFG grant:
Eleftheria Sarafidou

Project:
Global models of protoplanetary disks with all non ideal MHD effects taken into account and photoevaporation from X-rays from the central star.



Protoplanetary disk with MHD and photoevaporative wind (green area). The yellow lines represent the velocity streamlines of the wind, while the white are the magnetic field lines.

Paper on ArXiv !

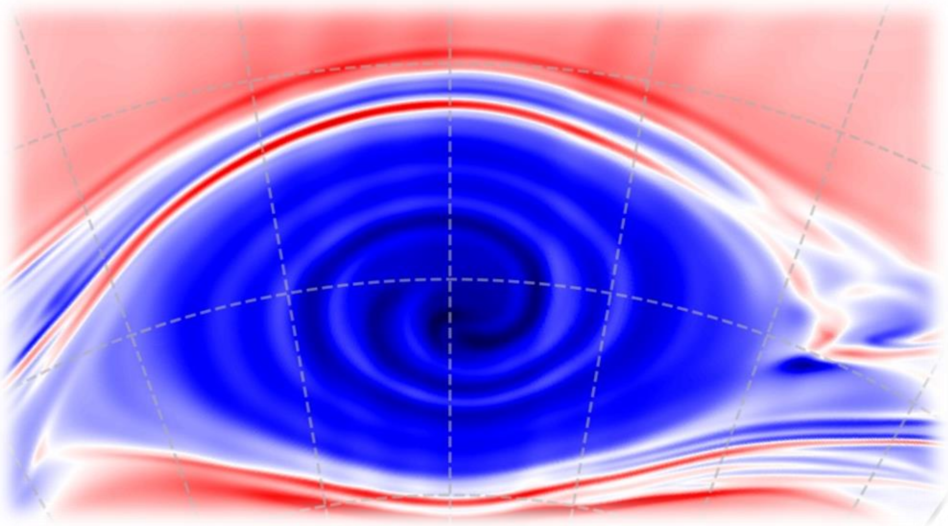
Outline

I. Introduction: planet formation assisted by vortices: a fundamental problem

II. The vertical stratification of bi-fluid (gas and dust) self-gravitating protoplanetary discs

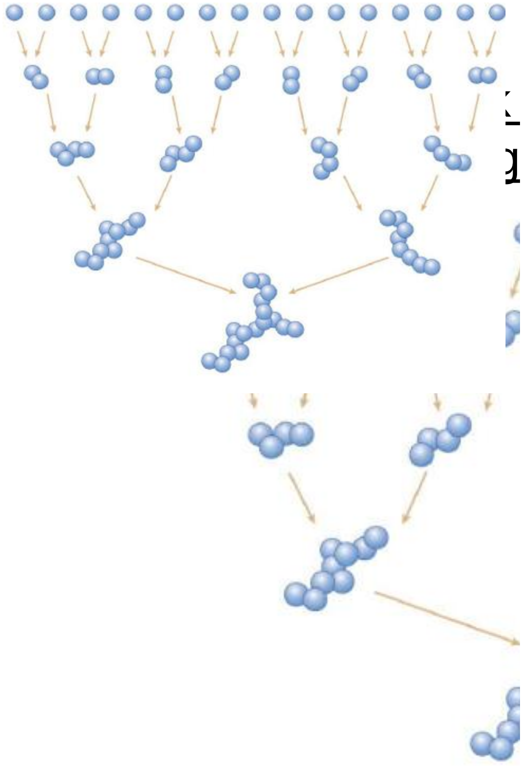
III. Self-gravity in 2D: smoothing length discarded and exact 2D kernel

I. Introduction: planet formation assisted by vortices: a fundamental problem



Core accretion model in a nutshell

1. Hit and stick



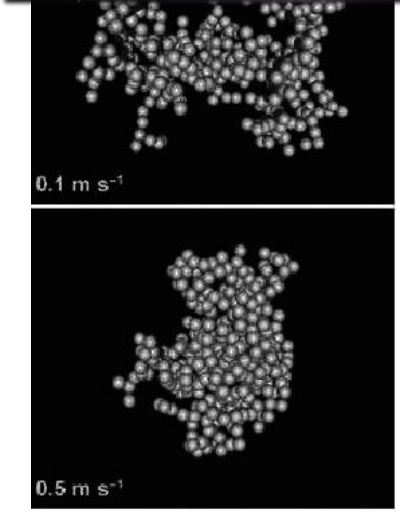
Blum, 2006

4. Giant planets formation: **gas envelope capture**

Credits: NASA/ESA/CSA, Jupiter ERS Team

few ~ Myr

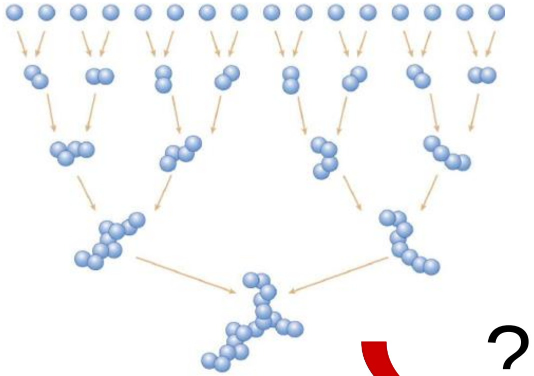
4. Giant planets formation: **gas envelope capture**



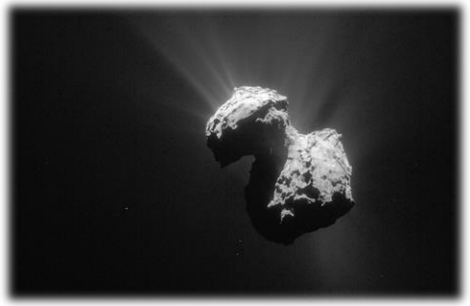
~100 km planetesimals in ~10 000 yr

Core accretion model in a nutshell

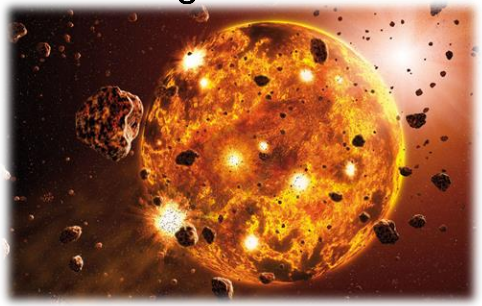
1. Hit and stick



2. Bodies **grow by collisions** until **planetesimals** formation



3. **Gravitational focusing** + **oligarchic** growth



4. Giant planets formation: **gas envelope capture**

???

Difficulties

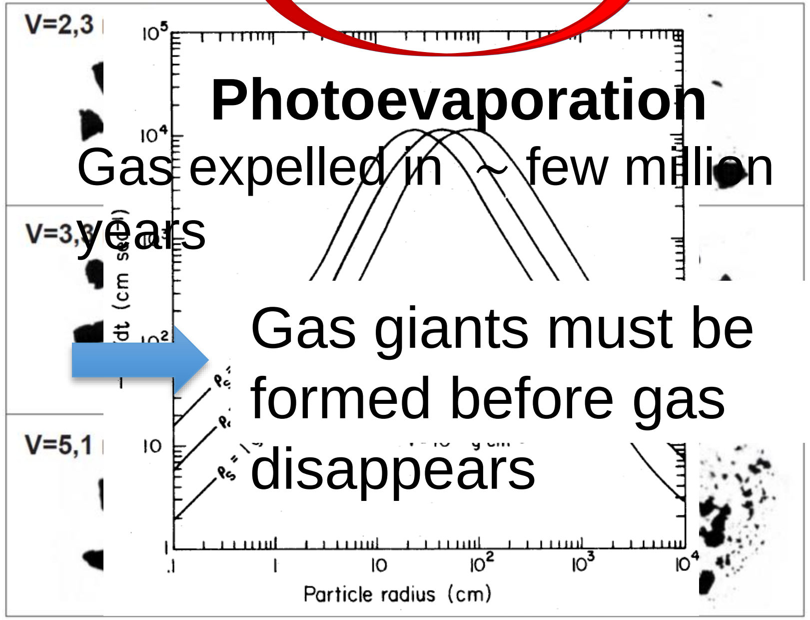
Radial drift (meter barrier) : 1m particles fall in only 85 years into the star (MMSN, 1 AU) !

Coagulation : cm aggregates don't stick and rather bounce (at m/s velocities)

Fragmentation during collisions and **erosion of planetesimals**

Possible mechanism

Streaming Instability
(Youdin & Goodman, 2005)



Photoevaporation

Gas expelled in ~ few million years

Gas giants must be formed before gas disappears

Güttler et al. 2010

Weidenschilling, 1977

Possible solution: anticyclonic vortices*

- Region of space where a flow is locally in rotation:
  $\omega = \vec{\nabla} \times \vec{v} \neq \vec{0}$

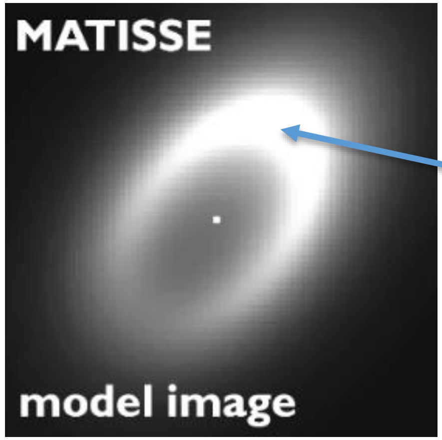
- Natural outcome of **hydrodynamical instabilities**: **Rossby Wave Instability (RWI)**, baroclinic, Vertical shear instability, edge of the gap carved by a massive planet …

- **Long-lived** structures (More than 1000 orbits around a star)

- Very efficient **dust traps** (local density increase x1000 in the vortex core)

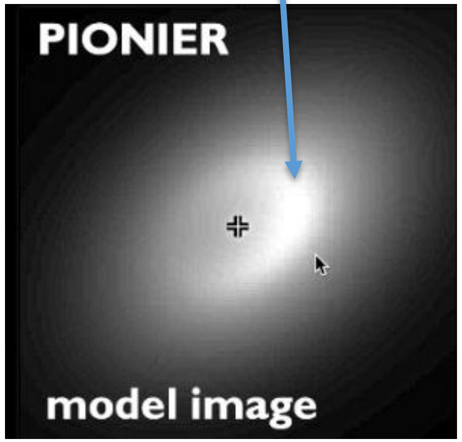
→ Planetesimal formation ?
Giant planet core formation ?

Possible indirect observations



(Varga et al. 2021)

It moved !



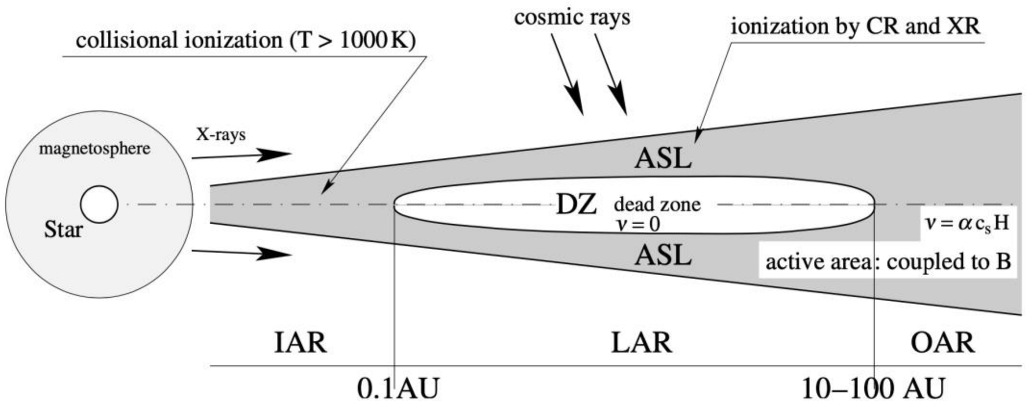
Asymmetry observed at **0.3 AU** from HD163296 star by MATISSE

* or pressure bumps

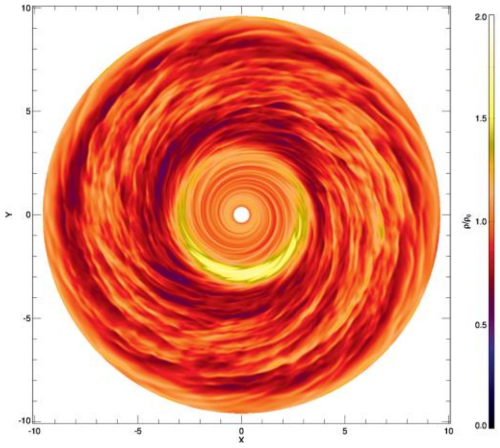
I. Introduction: planet formation assisted by vortices: a fundamental problem

How vortices form ?

- <u>Boundaries of the **dead zone**</u>
  Transition MRI active region (**α-turbulent**) and dead zone (**inviscid**)

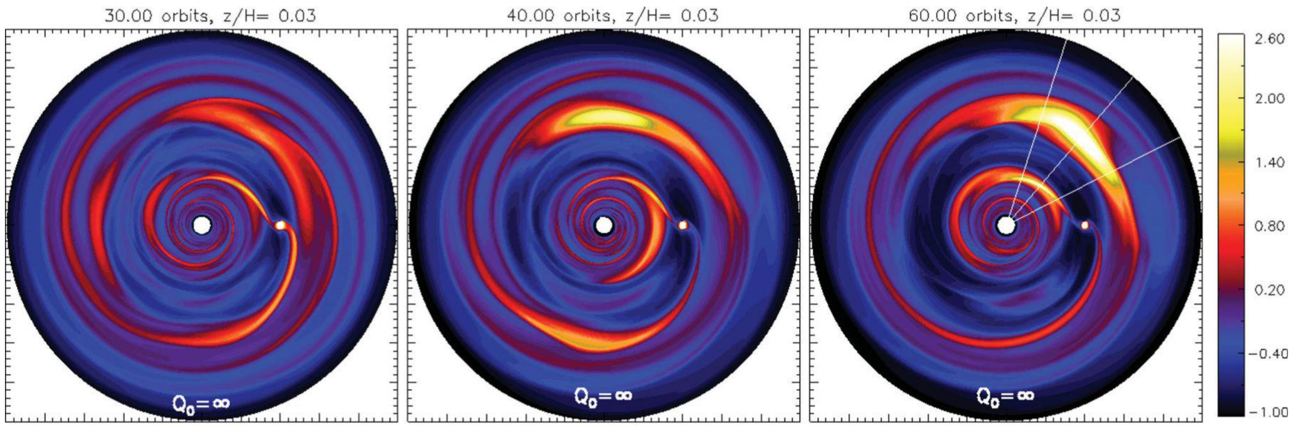


(Klahr & Brandner, 2006)



(Lyra & al. 2015)

- <u>At the edges of a **gap carved by a planet**</u>

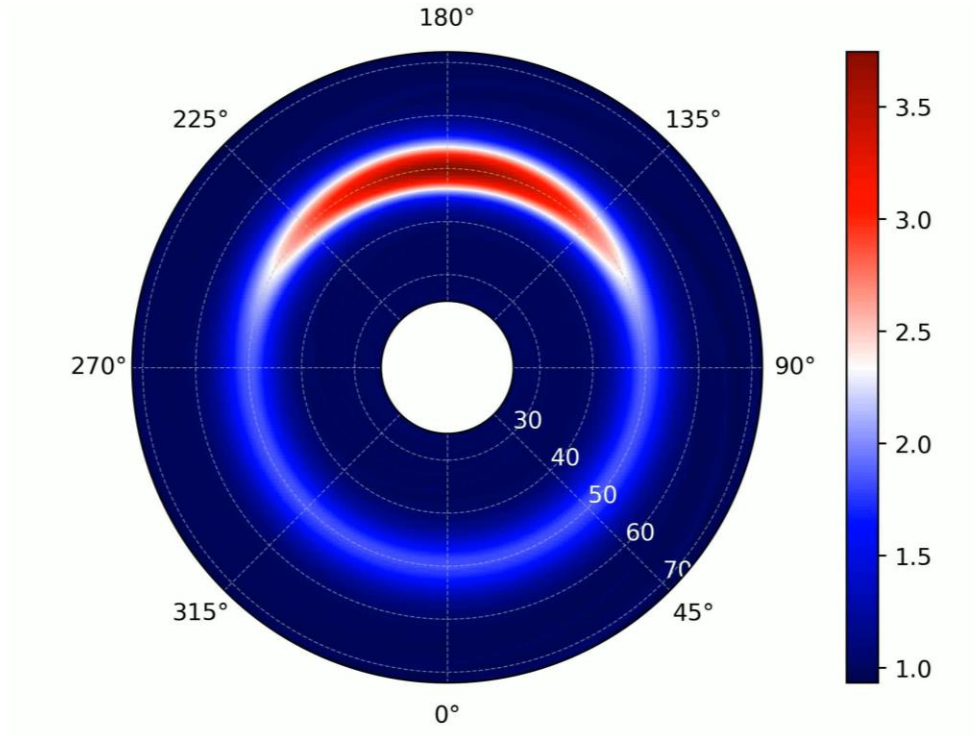


(Lin, 2012)

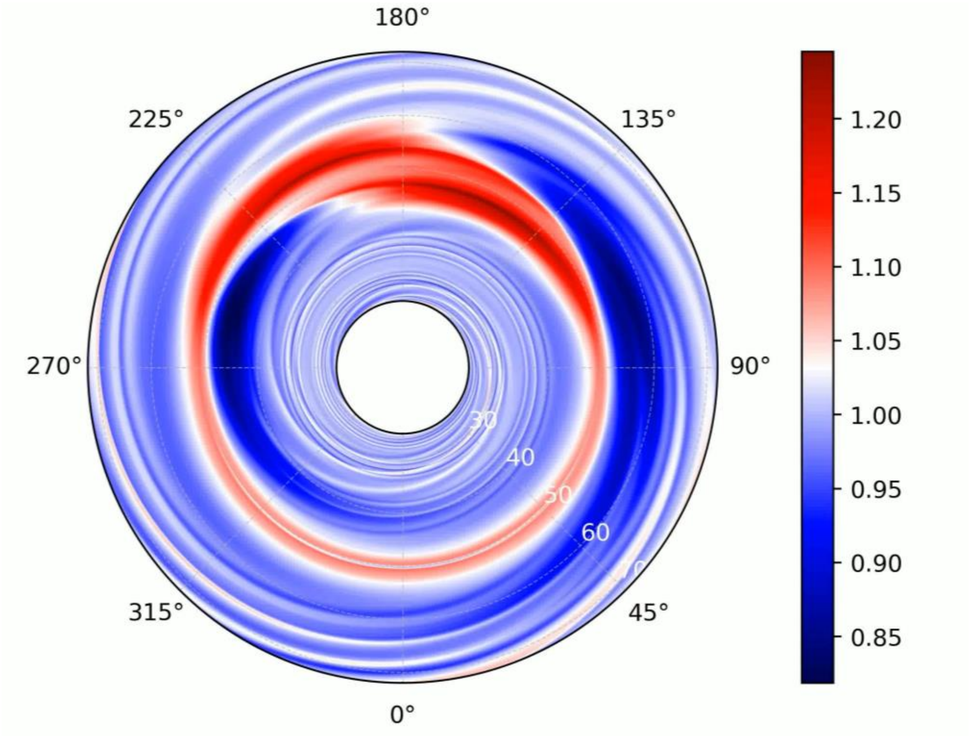
- More instabilities forming vortices : baroclinic (subcritical and convective-overstability), vertical shear instability, zombie vortex instability etc.

How vortices capture dust material ?

Gas density (normalised)



Particles density (normalised)

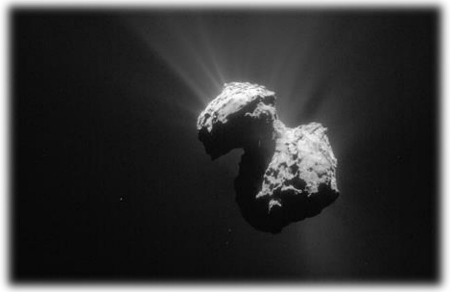


$t =$ 250.00 T_0 $(T_0 = 353$ yr)

St = 0.01 ; $r_p = 0.07$ cm ;
$\rho_{bulk} = 0.8\ g/cm^3$; Z = 0.001

2D Simulation (400 orbits) performed in Jean Zay cluster with *RoSSBi3D*

Self-gravity role



Self-gravity (SG) plays a key role :

- Cohesion of dust bind by gravity (planetesimal formation)
- Giant gas planet formation ?

Big problem

Despite favorable conditions (no dust feedback), huge dust-to-gas ratios, 2D simulations never showed a collapse.

Why ??? Programming problem ? **Theory ?**

How to prescribe SG in 2D (thin disc approximation) ?

Flat/Razor thin discs (H/r << 1)

Continuous overlap of infinitely flattened homoeiod shells

(Binney & Tremaine 2008)

Direct summation, closed form for the potential

(Durand 1953, Huré et al. 2008)

Thin disc (H/r small but not 0)

Average vertically the 3D SG force in a thin disc.

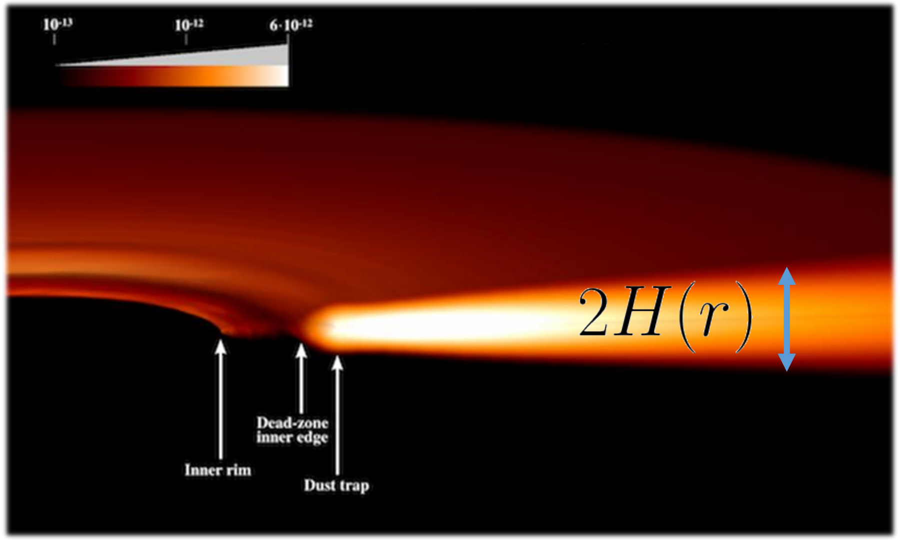
(Li et al. 2009, Müller et al. 2012, Rendon Restrepo & Barge 2023)

Resulting averaged force acts in the midplane of the disc.

Curiosity: If Universe was 2D gravity $\propto {}^{1}/_{r}$

How is computed SG in 2D simulations ?

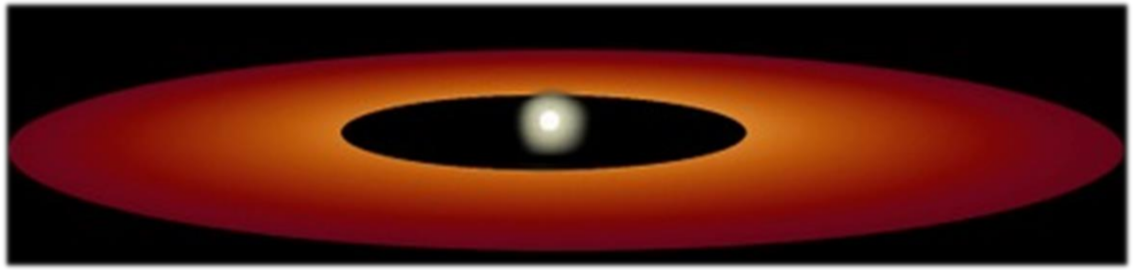
3D



(Flock et al. 2017)

$$\begin{cases} \rho = \rho(r, \theta, z) \\ P_{3D} = P_{3D}(r, \theta, z) \\ \vec{v}_{3D} \end{cases}$$

Thin disc (2D) approximation ($z/r < 1$)



Credit: NASA/JPL-Caltech/D. Watson

Quantities are vertically averaged and particularly SG forces:

$$f_{SG} = -\int \rho \nabla \Psi_{SG,3D}\, dz \quad \text{with} \quad \Psi_{SG,3D} = -\int_{disc} \frac{G\,\rho(\mathbf{r}')}{\sqrt{||\mathbf{r}-\mathbf{r}'||}} d^3\mathbf{r}'$$

How to prescribe SG in 2D (thin disc approximation) ?

Plummer potential - Account vertical thickness (and avoid singularities):

$$\Psi_{Plumm}(\boldsymbol{r}) = -\iint_{disc} \frac{\Sigma(\boldsymbol{r}')}{\sqrt{|\boldsymbol{r}-\boldsymbol{r}'|^2+\epsilon^2}}\, d^2\boldsymbol{r}'$$

Smoothing length (SL), ϵ, considered as a free parameter but analytical work converged to:

$$\epsilon_g/H_g = 0.6 - 1.2$$

(Huré et al. 2009, 2011, 2015; Müller et al. 2012)

Rendon Restrepo & Barge 2023

- Mid/short range SG interaction underestimated by 100%

A grav. collapse is impossible !

In agreement with removal of Newtonian behaviour in presence of softening

(Adams et al. 1989: Hockney & Eastwood 2021; Young & Clarke 2015)

- How to account for dust ?

Rendon Restrepo & Barge 2023

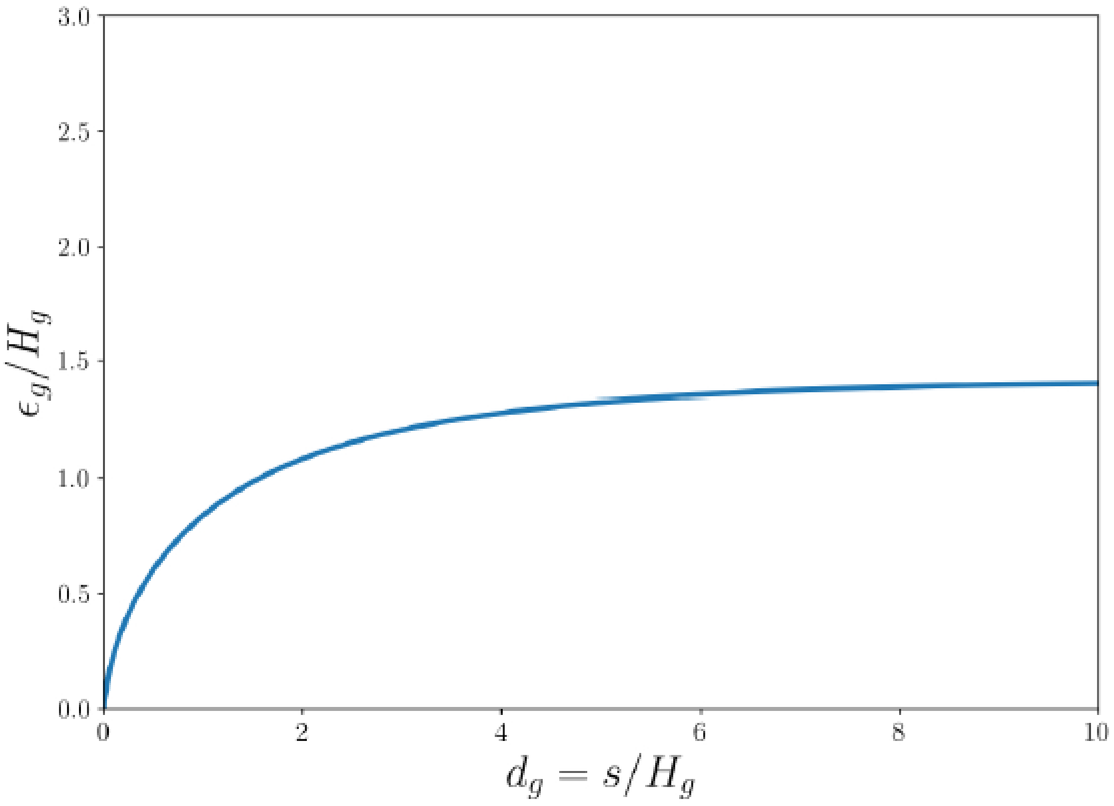
1. Introduced a Space Varying smoothing length

2. Generalized when dust present: 2 additionnal SL

Error decreased by factor 200 at short distances !

Dust SG can be underestimated by factor ≲1000

BUT …



3. Authors rushed on dust stratification assuming it Gaussian

Goal: find the exact SG kernel for thin disc simulations.

Two-step process:
1. Vertical hydrostatic equilibrium of the system
2. Resulting stratification is utilised as an input for vertically averaging all forces

So first we need to find the **vertical profile** of a **self-gravitating** protoplanetary disc made of **gas and dust**

II. The vertical stratification of bi-fluid (gas and dust) self-gravitating protoplanetary discs

$$\partial^2_{zz}\Phi_{disk} = 4\pi G\left(\rho_g + \rho_d\right)$$

$$\begin{cases} c_g^2\partial_z \ln\left(\rho_g\right) & = -\Omega_K^2 z - 2\pi G\left(\sigma_g + \sigma_d\right) \\ c_d^2\partial_z \ln\left(\rho_d/\rho_g\right) & = -\Omega_K^2 z - 2\pi G\left(\sigma_g + \sigma_d\right) \end{cases}$$

Assumptions: Isothermal vertically, Only Gas

Keplerian disc (No SG)

$$c_g^2 \partial_z \ln\left(\rho_g\right) = -\Omega_K^2 z$$

$$\rho_g = \frac{\Sigma}{\sqrt{2\pi} H_g} \exp\left[-\frac{1}{2}\left(z/H_g\right)^2\right]$$

with: $H_g = c_g/\Omega_K$

(Armitage 2015, 2022)

Massive disc (Strong SG)

$$c_g^2 \partial_z \ln\left(\rho_g\right) = -2\pi G \sigma_g$$

where: $\sigma(r,z) = \int_{-z}^{z} \rho(r,z')\, dz'$

$$\rho_g(\boldsymbol{r}, z) = \frac{\Sigma_g}{2 Q_g H_g} \operatorname{sech}^2\left(\frac{z}{Q_g H_g}\right)$$

with: $Q_g = \frac{c_g \Omega_K}{\pi G \Sigma_g}$ **Toomre's parameter**

(Spitzer 1942, Bertin & Lodato 1999)

Assumptions: Isothermal vertically, Only Gas

General case: From Keplerian to massive discs

$$c_g^2 \partial_z \ln(\rho_g) = -\Omega_K^2 z - 2\pi G \sigma_g$$

Approximate but accurate solution

$$\Rightarrow \quad \rho_g(\boldsymbol{r}, z) = \frac{\Sigma_g}{\sqrt{2\pi} H_g^{sg}} \exp\left(-\frac{1}{2}\left(z/H_g^{sg}\right)^2\right)$$

where: $H_g^{sg} = \sqrt{2/\pi}\, H_g\, f(Q_g)$ $\qquad f(x) = \frac{\pi}{4x}\left[\sqrt{1+\frac{8x^2}{\pi}} - 1\right]$

All information about SG hidden in the modified scale height ! (Bertin & Lodato 1999)

Problems:

1. In all known vertical profiles, self-gravity of gas and dust treated separately. Is that realistic ?

2. Is dust mass always negligible compared to gas ?

Observed low quantity of dust mass in discs might not adequately account for the mass of discovered exoplanets

3. There is no smooth solution from light (Keplerian) to massive discs including dust and gas

4. What is the Toomre’s parameter of gas/dust system ?

Vertical hydrostatic equilibrium of self-gravitating gas and dust disc

Dust embedded in turbulent gaseous environement

(Dubrulle et al. 1995)

$$
\begin{cases}
c_g^2 \partial_z \ln\left(\rho_g\right) = -\Omega_K^2 z - 2\pi G\left(\sigma_g + \sigma_d\right) \\
c_d^2 \partial_z \ln\left(\rho_d / \rho_g\right) = -\Omega_K^2 z - 2\pi G\left(\sigma_g + \sigma_d\right)
\end{cases}
$$

Vertical component star gravity

Vertical component SG gas AND dust

For lovers of Maths

Both equations reduced into a unique modified Liouville equation

Fortunately, we found 4 new exact solutions for gas AND dust !

And the **general** case ?

Approach of Bertin & Lodato 1999

=

“Biased" Gaussian stratification, where all **SG information is incorporated into a modified scale** height, which is naturally Toomre’s parameter dependent.

We generalized their procedure to a self-gravitating gas/dust disc

And the **general** case ?

Both fluids experience an equal gravitational influence from the star and from their combined mass distributions

$$\begin{cases} \rho_g(\boldsymbol{r}, z) = \frac{\Sigma_g}{\sqrt{2\pi} H_g^{sg}} \exp\left[-\frac{1}{2}\left(z/H_g^{sg}\right)^2\right] \\ \rho_d(\boldsymbol{r}, z) = \frac{\Sigma_d}{\sqrt{2\pi} H_d^{sg}} \exp\left[-\frac{1}{2}\left(z/H_d^{sg}\right)^2\right] \end{cases}$$

with:

$$\begin{cases} H_g^{sg} = \sqrt{\frac{2}{\pi}} H_g f(\tilde{Q}) \\ H_d^{sg} = \sqrt{\frac{2}{\pi}} \frac{\xi}{\sqrt{\xi^2+1}} H_d f(\tilde{Q}) \\ \tilde{Q} = \left(\frac{1}{Q_g} + \frac{\sqrt{\xi^2+1}}{\xi}\frac{1}{Q_d}\right)^{-1} \\ f(\tilde{Q}) = \frac{\pi}{4\tilde{Q}}\left[\sqrt{1 + \frac{8\tilde{Q}^2}{\pi}} - 1\right] \end{cases}$$

Generalized Toomre's parameter for a system of gas and dust

Dust is sustained in a turbulent gaseous environment

$\xi = c_g / c_{d,mid}$ and $c_{d,mid}^2 = \kappa_t / \tau_f$

Other implications ?

Discs accrete material ~ 1st Myr → Possible source of turbulence (MRI, GI, VSI, SBI)

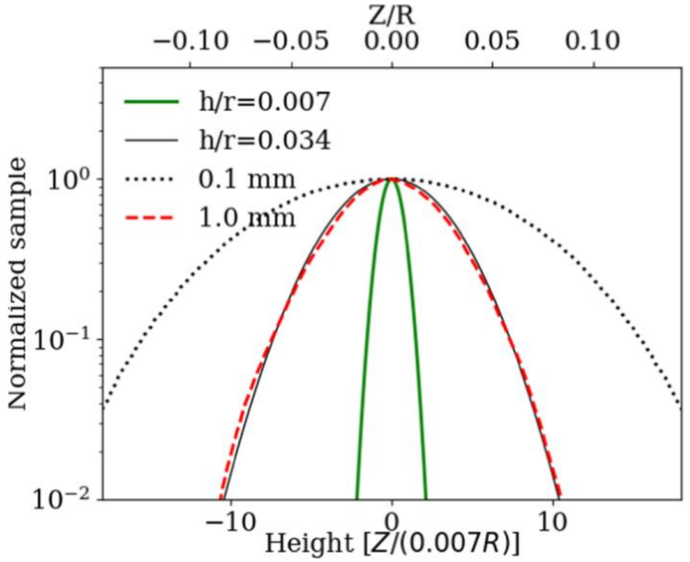
Gas turbulence cannot be measured → Instead we use dust

$$H_{\rm d} = \sqrt{\frac{\tilde{\alpha}}{\tilde{\alpha}+{\rm St}}} H_{\rm g}$$

(Dubrulle et al. 1995; Weber et al. 2019)

Simulations of VSI show strong levels of turbulence/mixing

(One possible solution is that other insta. suppress VSI)



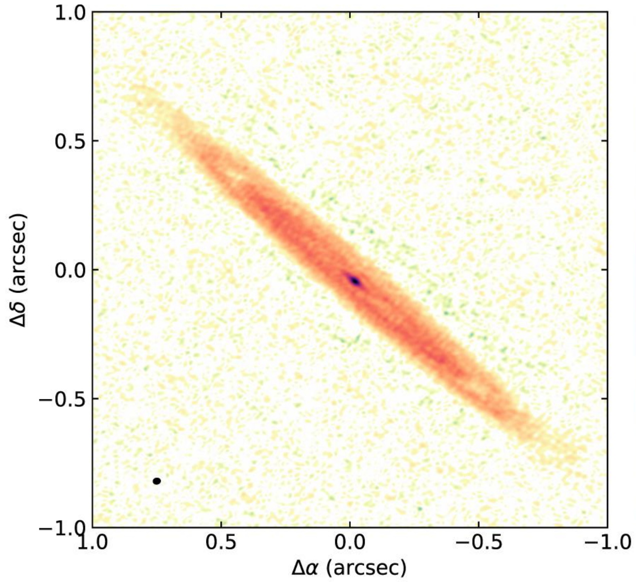
Flock et al. (2020)

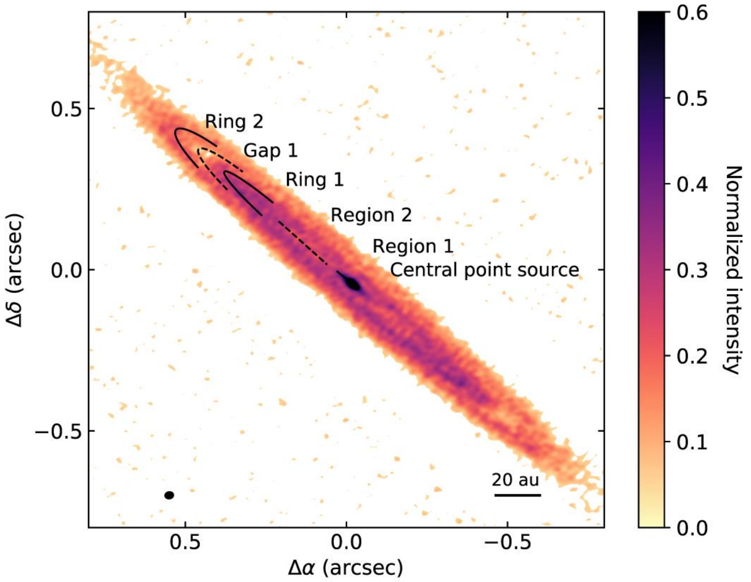
Green Pinte et al. - HL Tau

Other implications ?

Observations show very thin dust layers, equivalent to $\alpha \sim 10^{-5} - 10^{-4}$

Pinte (2016) , Villenave et al. (2020, 2022)





Other explanation for thin dust layer and strong accretion:
WINDS: remove angular momentum (= accretion) and don’t add turbulence in the midplane (=settled dust)

Other implications ?

Baehr & Zhu 2021 showed thanks to simulations of gas and dust that discrepancy between thin dust layers and strong accretion can be explained with **gravito-turbulence** and that **dust contribution to SG cannot be neglected**.

Model will help to understand finely the high settling of dust and strong accretion of gas.

I **suspect** that the vertical Schmidt number **(anisotropy turbulence)** depends on the Generalized Toomre's parameter

This needs to be checked thanks to Shearing BOX simulations ! (In process)

Soon: shearing box simulations with dust/gas SG and spectral (FFT) methods

Do I expect to find the theoretical stratification ?

Not really !

Everyone forgets about this term in full periodic SBOX

$$\Delta\Phi_g = 4\pi G\left(\rho - \bar{\rho}\right)$$

From a gravitational point of view:

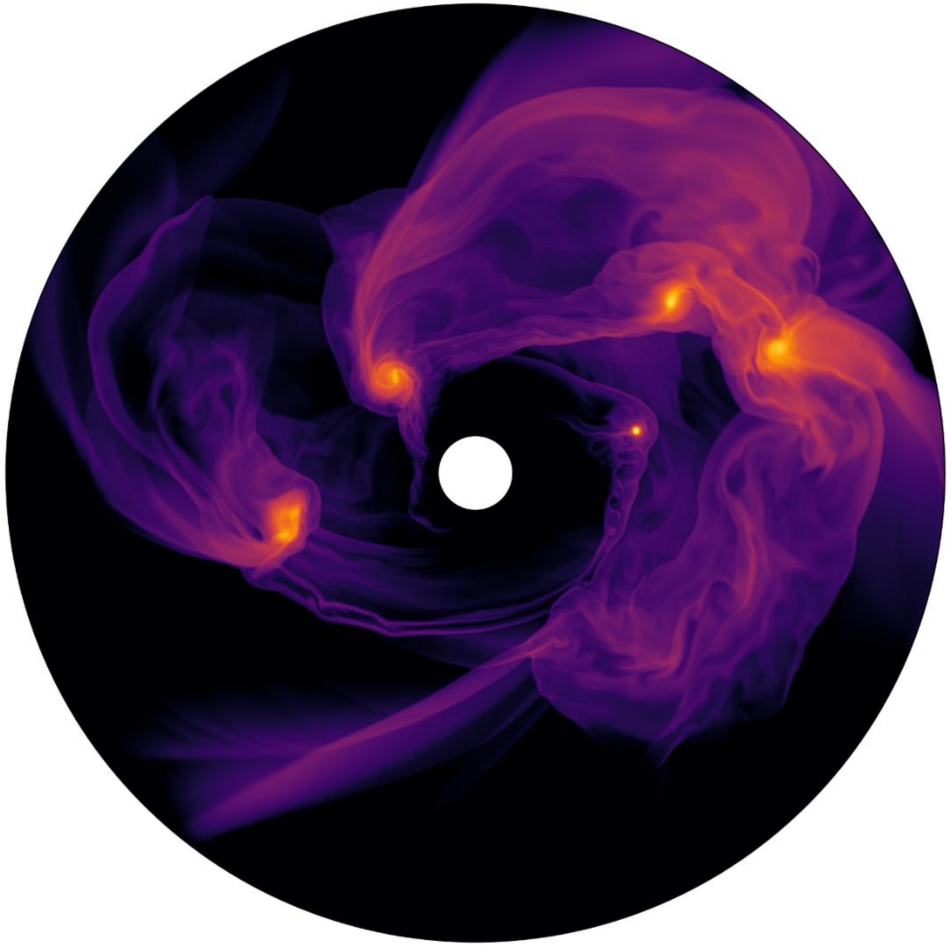
- There is less mass in the box !
- In average the total mass is 0.

III. Self-gravity in 2D: smoothing length discarded and exact 2D kernel

Self-gravity in thin protoplanetary discs:

1. The smoothing-length discarded by the exact 2D self-gravity kernel

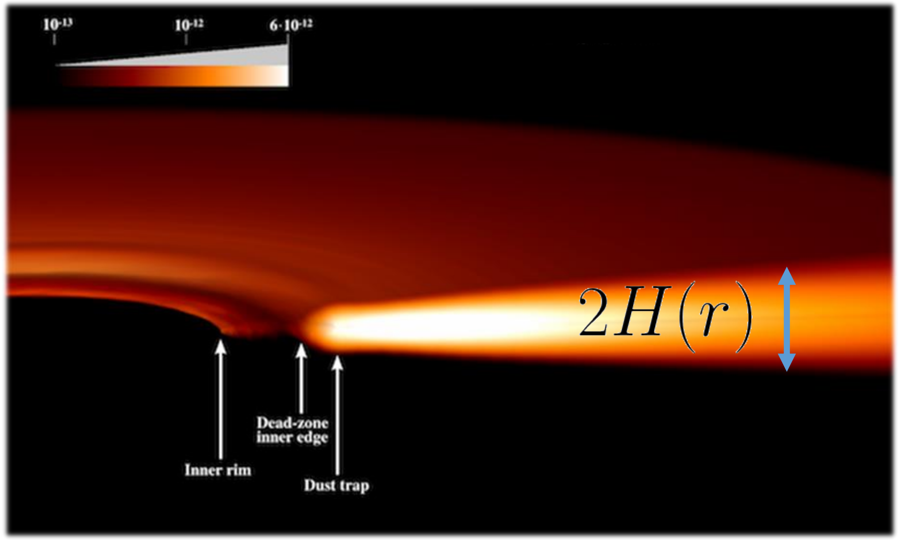
S. Rendon Restrepo[1] ⋆, T. Rometsch[2] ⋆⋆, O. Gressel[1], and U. Ziegler[1]



Not yet in ArXiv but main results shown here

How is computed SG in 2D simulations ?

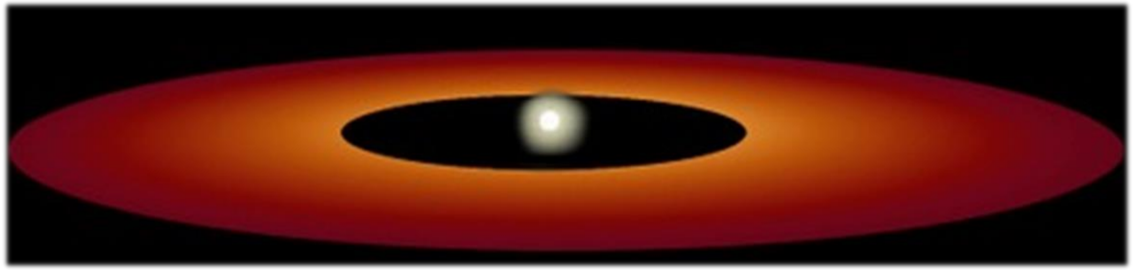
3D



(Flock et al. 2017)

$$\begin{cases} \rho = \rho(r,\theta,z) \\ P_{3D} = P_{3D}(r,\theta,z) \\ \vec{v}_{3D} \end{cases}$$

Thin disc (2D) approximation ($z/r < 1$)



Credit: NASA/JPL-Caltech/D. Watson

Quantities are vertically averaged and particularly SG forces:

$$f_{SG} = -\int \rho \nabla \Psi_{SG,3D}\, dz \quad \text{with} \quad \Psi_{SG,3D} = -\int_{disc} \frac{G\,\rho(\mathbf{r}')}{\sqrt{||\mathbf{r}-\mathbf{r}'||}}\, d^3\mathbf{r}'$$

In a Gaussian stratified disc, all SG contributions can be summarized in:

$$\boldsymbol{f}_{2D}^{a\to b}(\boldsymbol{r}) = -\Sigma_b \iint_{disc} \Sigma_a \, \mathrm{K}_{ab} \, \boldsymbol{e}_s \, d^2\boldsymbol{r}'$$

where:

$$\mathrm{K}_{ab} = \frac{1}{2\pi} \frac{s}{H_a^{sg} H_b^{sg}} \int_{z,z'=-\infty}^{+\infty} \frac{e^{-\frac{1}{2}\left(z/H_b^{sg}\right)^2} e^{-\frac{1}{2}\left(z'/H_a^{sg}\right)^2}}{\left(s^2 + (z - z')^2\right)^{3/2}} \, dz \, dz'$$

$$s = \|\boldsymbol{r} - \boldsymbol{r}'\|$$

Is the **self-gravity force kernel**.

(Müller et al. 2012, Rendon Restrepo & Barge 2023)

Closed form of the integral = very difficult !

That's why it was approximated with a Plummer potential:

$$\delta\Psi_{Plumm} = \frac{1}{s^2 + \epsilon^2}$$

$$\epsilon \simeq 0.6 H_g$$

We rediscovered and improved (Li et al. 2009):

$$K_{ab} = \frac{1}{\sqrt{\pi}}\left(H_{ab}^{sg}\right)^{-2}\frac{d_{ab}}{8}\exp\left(\frac{d_{ab}^2}{8}\right)\left[K_1\left(\frac{d_{ab}^2}{8}\right)-K_0\left(\frac{d_{ab}^2}{8}\right)\right]$$

where:

- r.m.s scale height: $H_{ab}^{sg} = \sqrt{\frac{H_a^{sg2}+H_b^{sg2}}{2}}$

Solves symmetry problem (respects 3rd Newton's law)

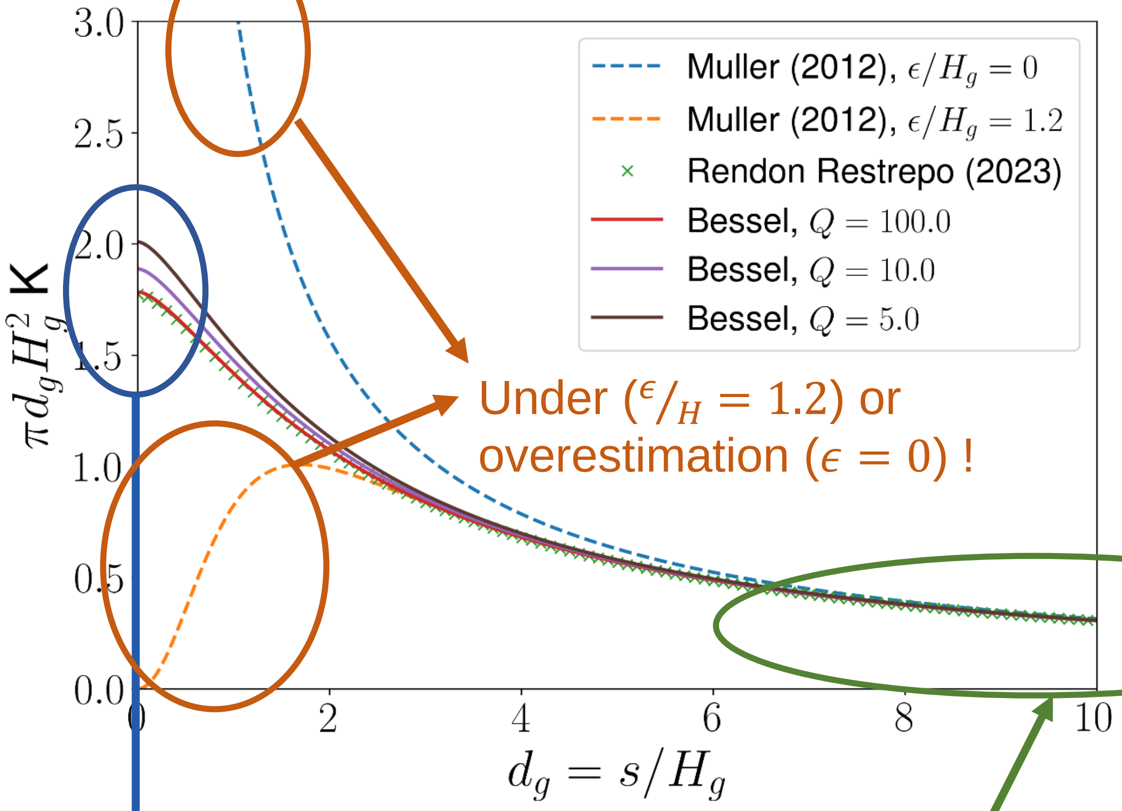
$$d_{ab} = s/H_{ab}^{sg}$$

New

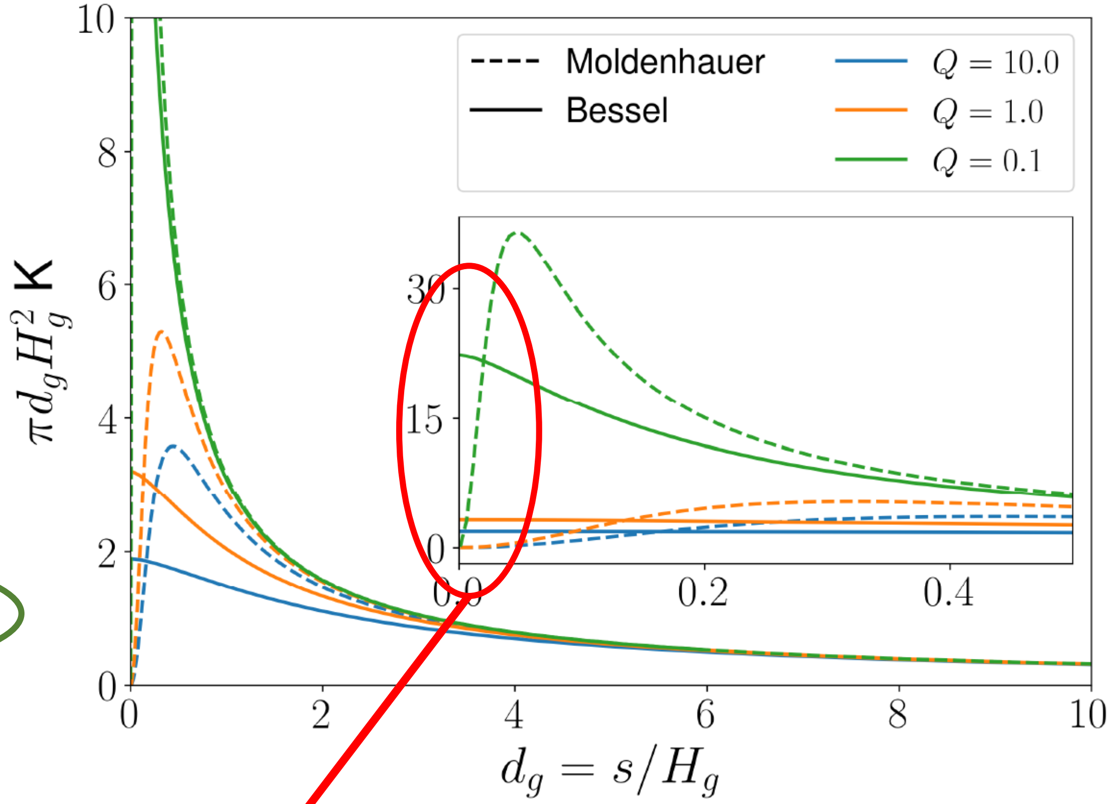
- Bi-fluid analysis
- Incorporates how the SG of both components affects their vertical density profile
- Compatible with FFT methods (not contemplated by Li et al. 2009)
- Transition from light to massive discs
- **And respects Newtonian character of gravity !!!**

III. Self-gravity in 2D: smoothing length discarded and exact 2D kernel

Normalised self-gravity kernels with respect to distance



Under ($^{\epsilon}/_{H} = 1.2$) or overestimation ($\epsilon = 0$) !



Light discs

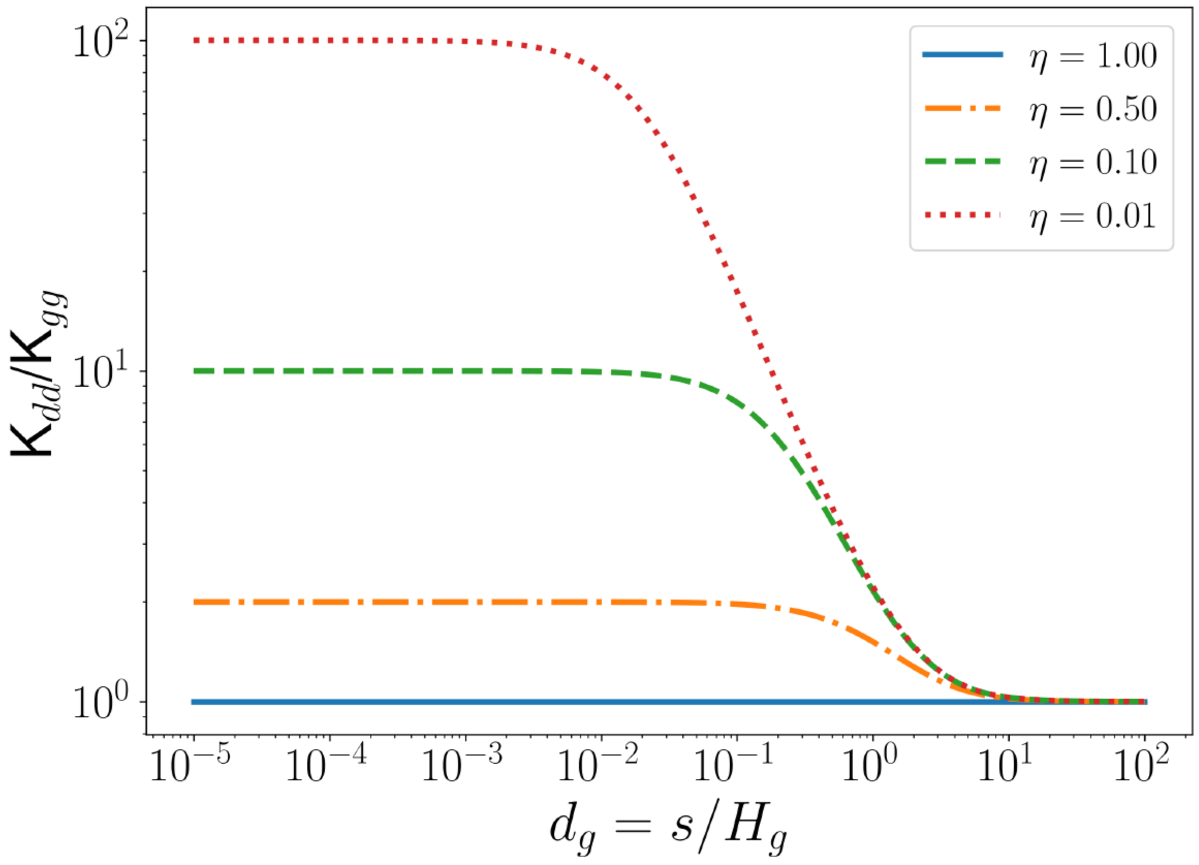
At large distances gravity retrieves its 3D behaviour

Massive discs

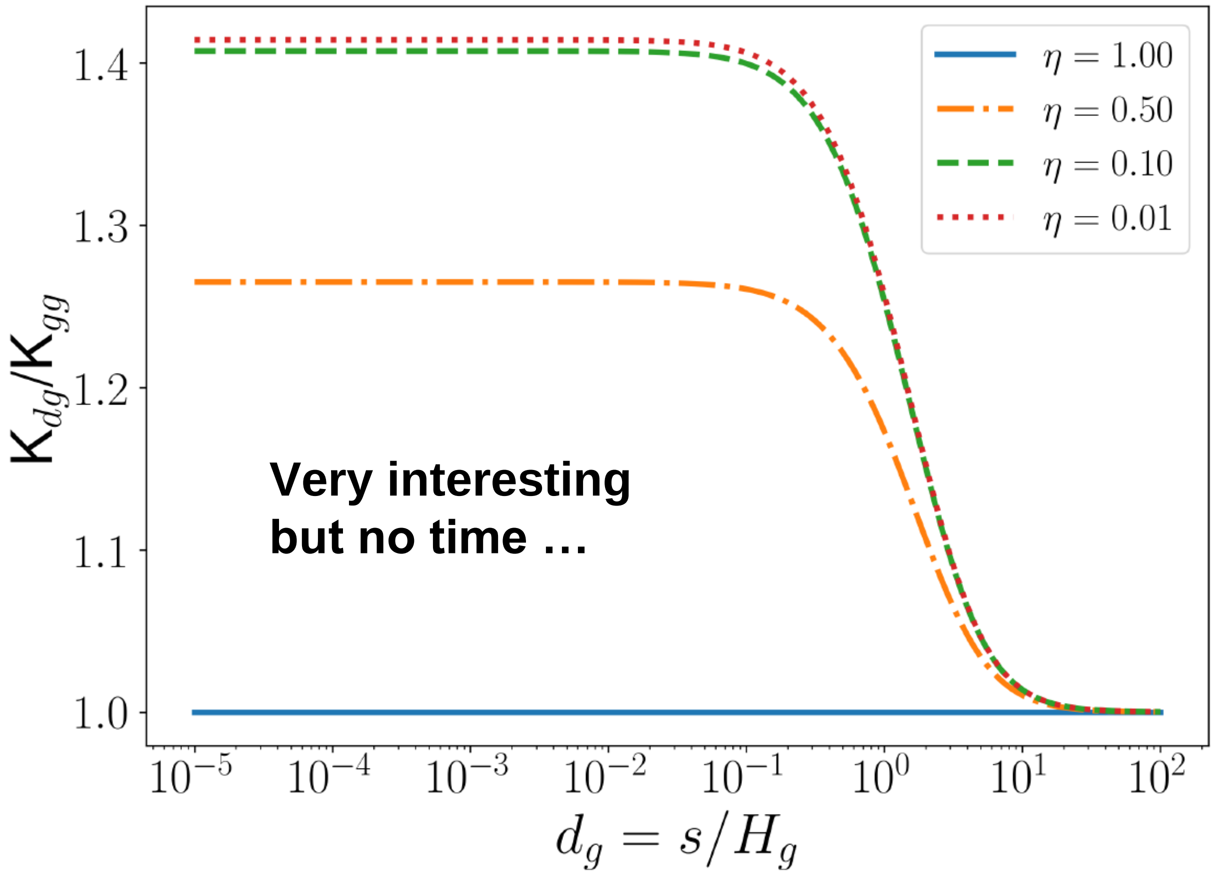
At short distances gravity behaves as in a 2D Universe

Only Bessel kernel permits fragmentation at infinitesimal distances. Unnoticed mecanism

Normalised kernels associated with dust with respect to distance for different dust-to-gas scale heights, $\eta = {}^{H_d}/_{H_g}$



Dust-dust kernel



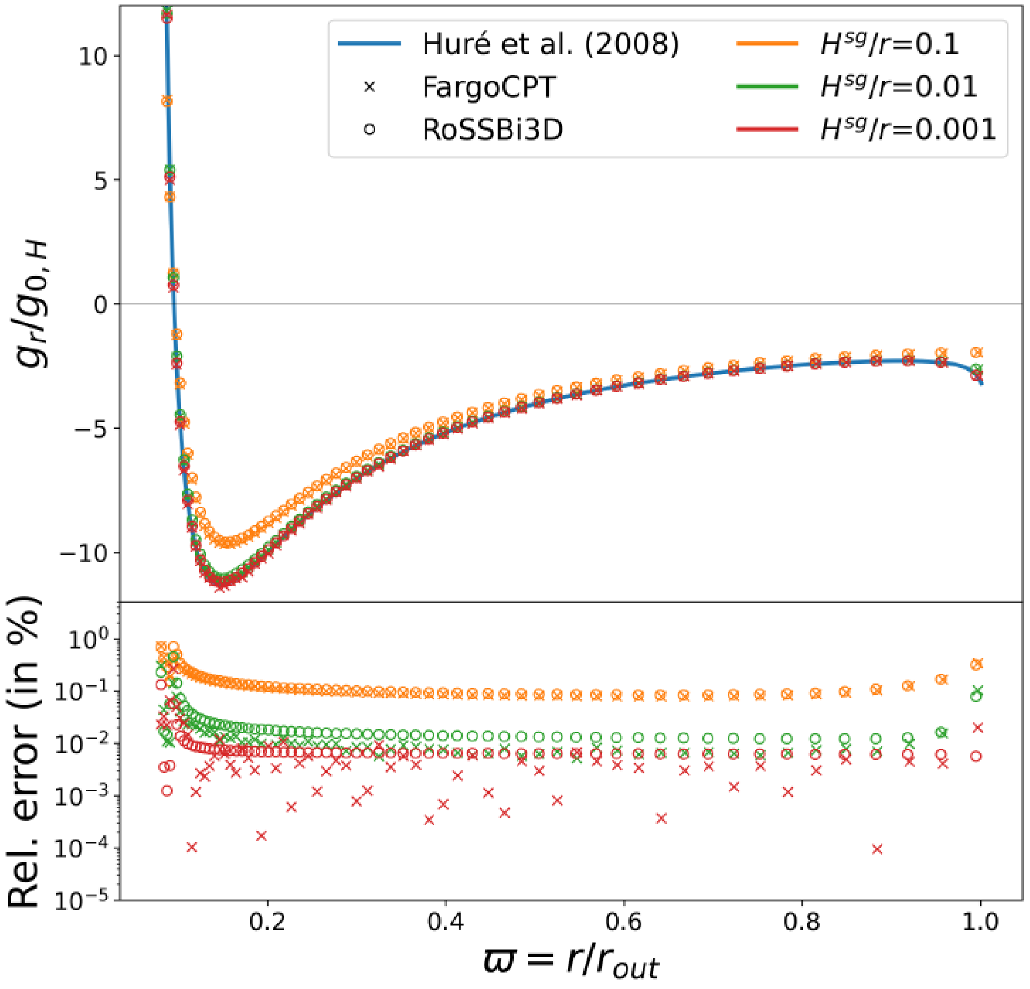
Dust-gas kernel

How to be sure it's the correct SG kernel for 2D ?

- We perform analytical benchmarks (not showed here, too much maths for today)

- 2D numerical benchmarks in the limit of razor-thin discs with exact solutions

- 2D/3D dynamical benchmarks (in process)



What consequences for planet formation theories ?

Gravitational Instability **(GI)**

Disc cools down

→ Pressure decreases

→ Gravity overcomes pressure and tidal forces

Turbulence
+
→ Spiral formation
+
(Fragmentation if fast cooling)

- Gravitoturbulence **(GI):** $1 < Q \lesssim 1.5$

$$Q = \frac{c_s \Omega}{\pi G \Sigma}$$

Depending on β (cooling) we get $\alpha \sim 0.01 - 0.1$

What consequences for planet formation theories ?

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→ Pressure decreases

→ Gravity overcomes pressure and tidal forces

Turbulence
+
→ Spiral formation
+
(Fragmentation if fast cooling)

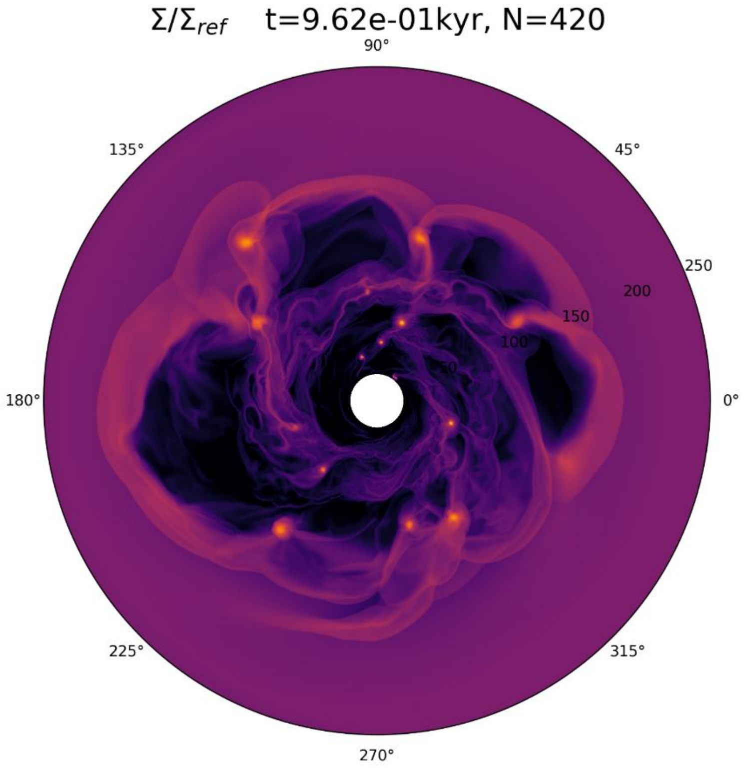
HR simulations with FargoCPT

(Rometsch et al. 2024)

- $(N_r, N_\theta) = (1400,\ 4800)$ (Resolve $Q_g\ H_g$)
- $r\ \in [20, 250]$ AU
- Müller et al. 2012 and Bessel kernel
- β −cooling = [2, 8] (Gammie 2001)
- Disc cools to 0 K
- No indirect term

$\beta = 2$ → fragmentation expected

$\epsilon / H = 0$

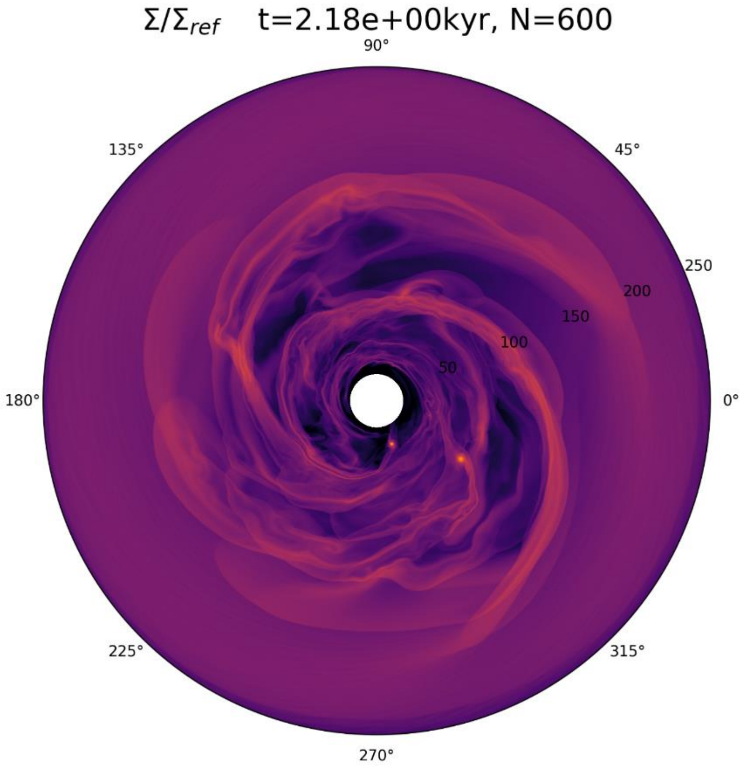


Formation excess ?

Object too massive ?

$Max(\Sigma/\Sigma_{ref}) = 7300$

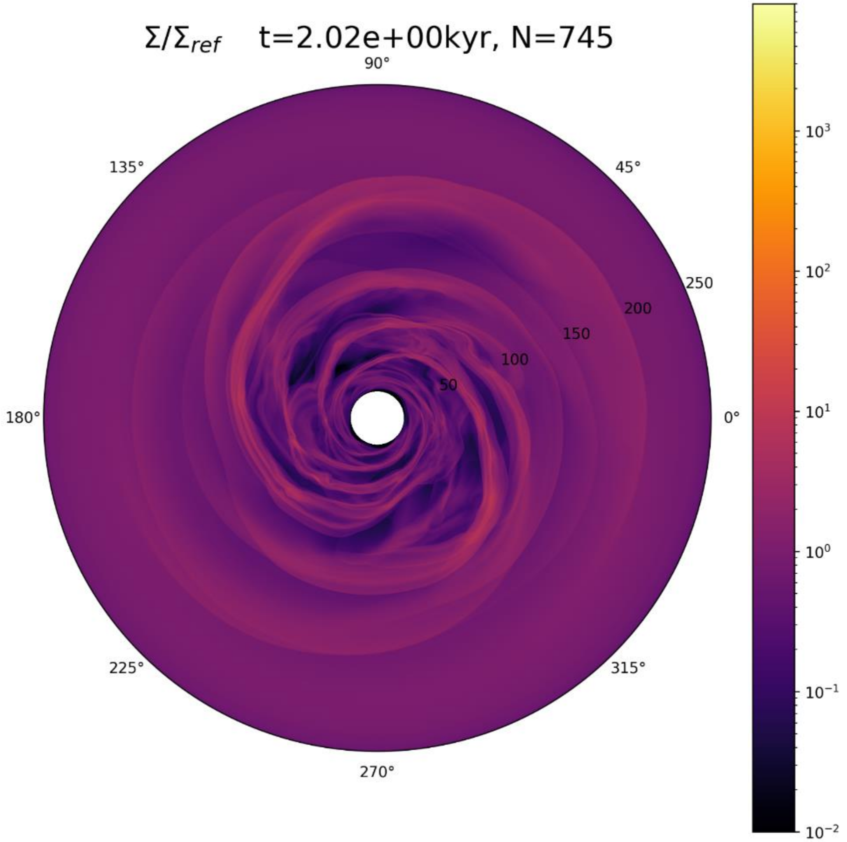
Bessel kernel that we propose



Few objects bound by gravity

$Max(\Sigma/\Sigma_{ref}) = 760$

$\epsilon / H = 1.2$

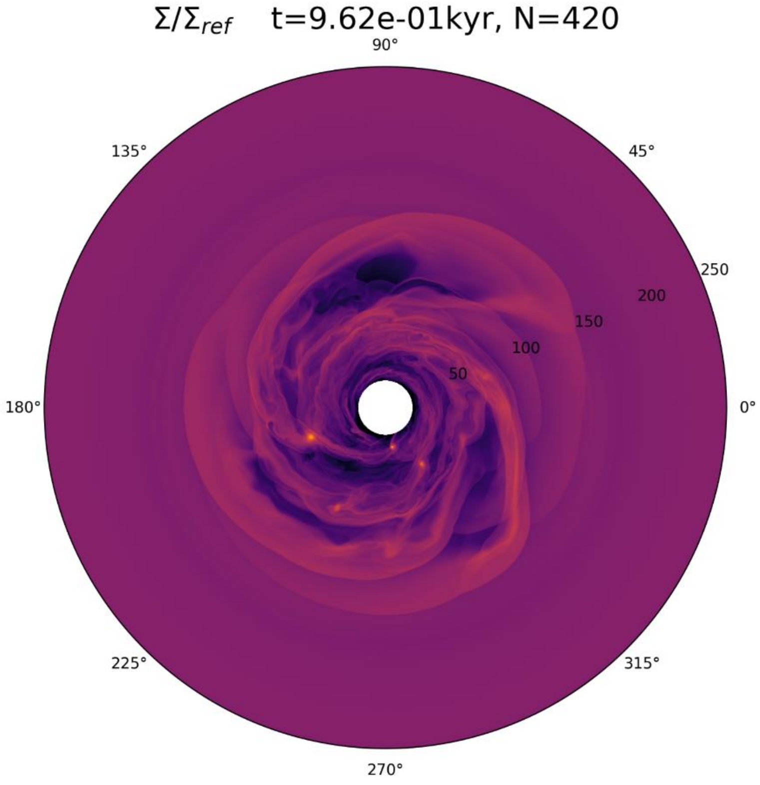


No formation of objects bound by gravity

$Max(\Sigma/\Sigma_{ref}) = 12.6$

$\beta = 8$ → gravito-turbulence (no fragmentation expected)

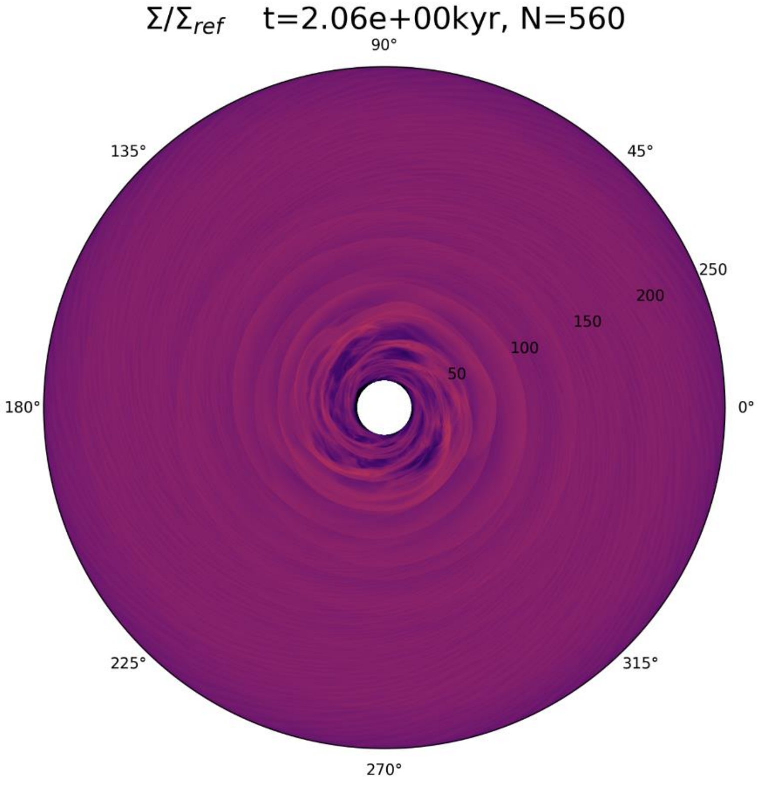
$\epsilon / H = 0$



Still fragments !

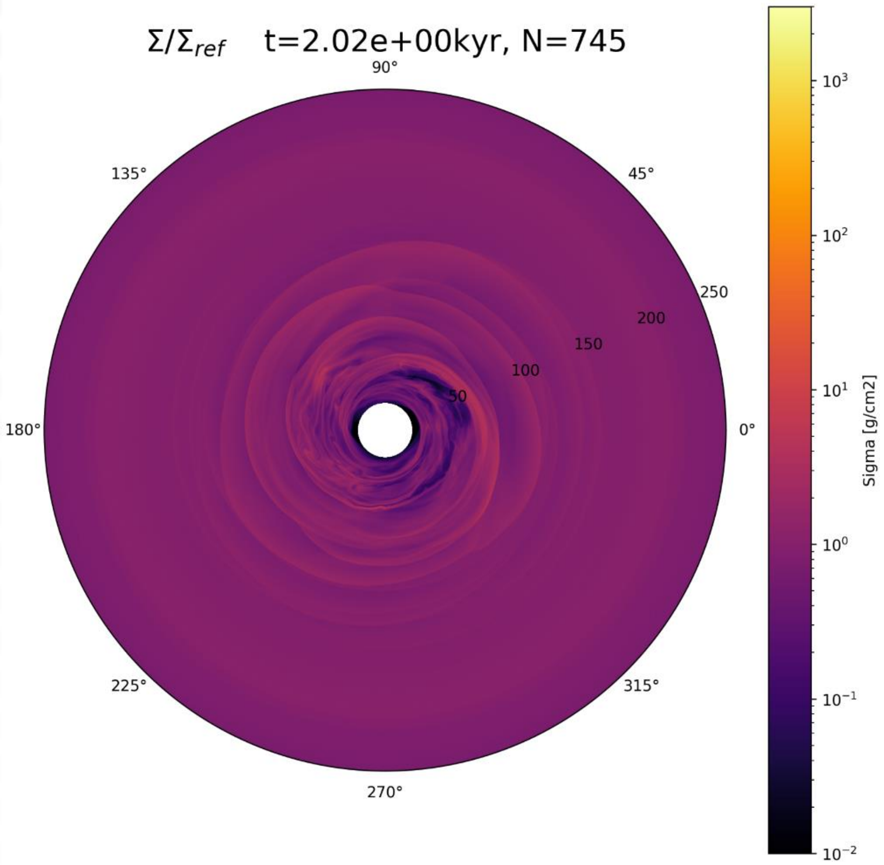
$$Max(\Sigma/\Sigma_{ref}) = 1428$$

Bessel kernel that we propose



$$Max(\Sigma/\Sigma_{ref}) = 5.41$$

$\epsilon / H = 1.2$



No formation of objects bound by gravity

$$Max(\Sigma/\Sigma_{ref}) = 5.32$$

Bessel kernel

- Fragmentation occurs at lower Q_g values: consistent with the fact that gravity is « diluted » vertically (Kim et al. 2002; Wang et al. 2010; Baehr et al. 2017)

- This kernel may solve many problems related to fragmentation **(in process)** :

1. Estimation of the β_{critic}
2. Numerical convergence problems encountered in 2D (Young & Clarke 2015)
3. Is fragmentation stochastic ? (Paardekooper 2012)

- **Are the formed clumps still to massive ? (GI usually forms brown dwarfs)**

Bessel kernel

- We need to update the dispersion relation of fragmentation of discs with the Bessel kernel (But expect the threshold to be slightly smaller than standard value)

- Really suspect that the 2D Poisson’s equation in 2D is simply wrong and leads to an **overestimation of SG** !

- When SL=0, **overestimation of SG !**

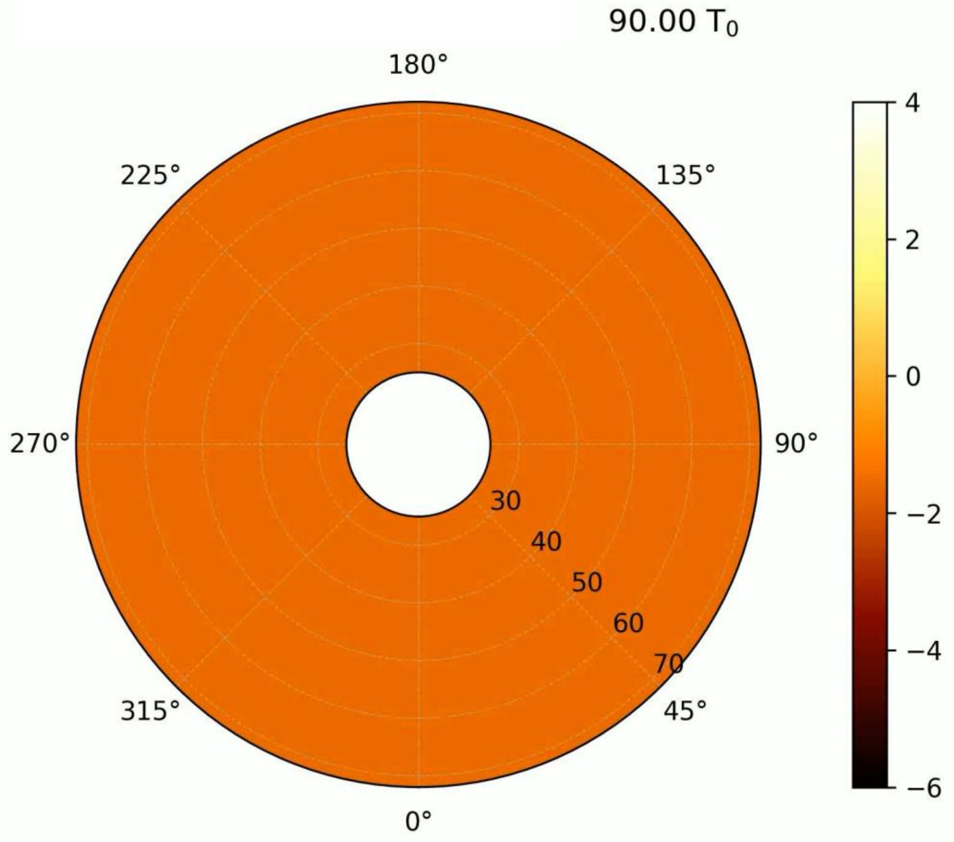
In collaboration with Thomas Rometsch, Oliver Gressel and Udo Ziegler

Initial problem: How corrections affect the vortex scenario ?

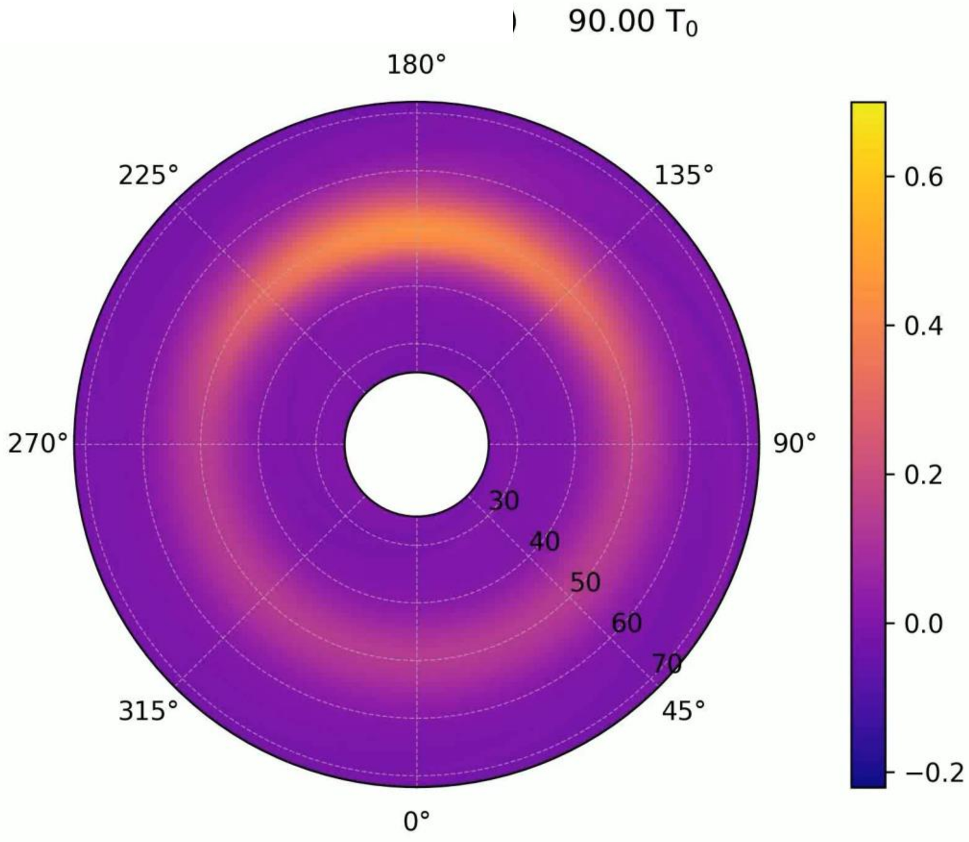
$St \approx 0.5$, r= 50 AU, $\eta = {}^{H_g}/_{H_d} = 50$,
Z=0.1 (high on purpose), resolution: H/30

Initial state: Gaussian vortex
+ uniform dust distribution

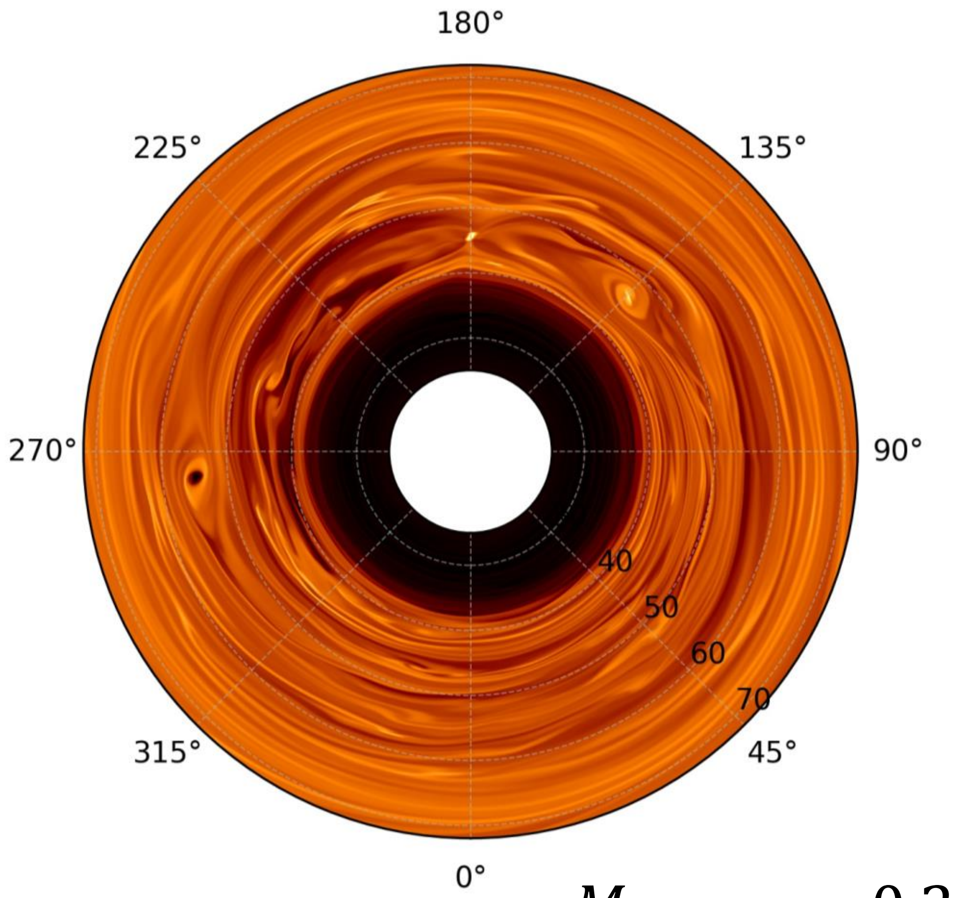
$\ln\left(\Sigma_d/\Sigma_{g,0}\right)$



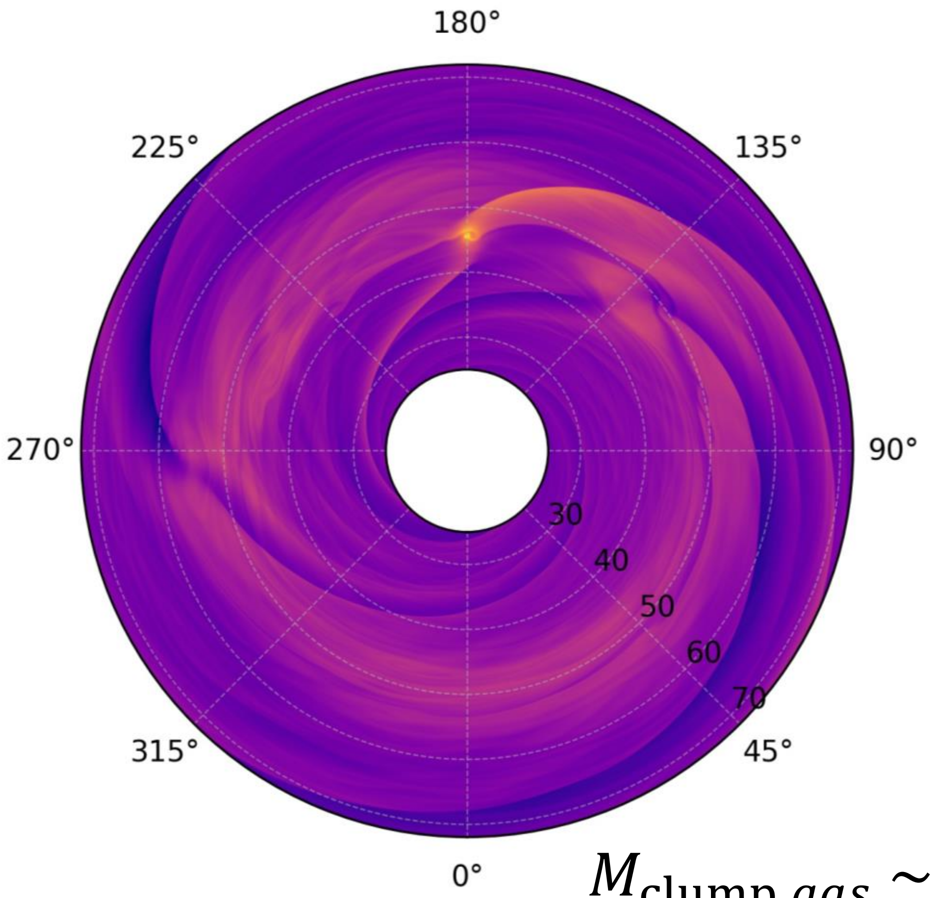
$\ln\left(\Sigma_g/\Sigma_{g,0}\right)$



Final state



$M_{clump,d} \sim 0.2\ M_J$



$M_{\mathrm{clump},gas} \sim 0.08\ M_J$

- Gas and dust in horseshoe motion
- Lindblad resonances ?
- Migration ?

To take with a grain of salt (old simulations):

- No dust back-reaction
- No dust diffusion

Take-home messages

- Grav. Collapse inside vortices may be possible thanks to the correct **SG prescription !!!**

- Vertical stratification of a **self-gravitating protoplanetary** disc made of **gas and dust.**

- Correct definition of the Toomre’s parameter of a bi-fluid system

- Analytical **kernel** for SG in 2D simulations: seems to correct all issues inherent to a Plummer potential: **symmetry**, **underestimation /overestimation** of SG at short distances, accounts **for self-consistent stratification of gas and dust**.

Take-home messages

- It will be difficult to convince astrophysicists that the simplistic 2D Plummer potential:

$$\delta\Psi_{Plumm} = \frac{1}{s^2 + \epsilon^2} \qquad s = \|\boldsymbol{r} - \boldsymbol{r}'\|$$

Should be replaced by:

$$\mathrm{K}_{ab} = \frac{1}{\sqrt{\pi}}\left(H_{ab}^{sg}\right)^{-2}\frac{d_{ab}}{8}\exp\left(\frac{d_{ab}^2}{8}\right)\left[K_1\left(\frac{d_{ab}^2}{8}\right) - K_0\left(\frac{d_{ab}^2}{8}\right)\right]$$

Solution: $$\epsilon^2 = \frac{1}{\frac{1}{\sqrt{\pi}}\left(H_{ab}^{sg}\right)^{-2}\frac{d_{ab}}{8}\exp\left(\frac{d_{ab}^2}{8}\right)\left[K_1\left(\frac{d_{ab}^2}{8}\right) - K_0\left(\frac{d_{ab}^2}{8}\right)\right]} - s^2$$

Perspectives

- Finish the three papers 😅

- Use the kernel in an Astrophysical context of massive discs (Elias 2-27, IM Lupi and GM Aur)

- Discs with a thin and massive dust layers are the most unstables and the instability is dust driven. Formation of terrestrial planets (Longarini et al. 2023). Our prescription is perfect for probing this statement thanks to 2D global simulations.

- Baehr & Zhu 2021 found Vertical Schmidt numbers of ~ 200. Can we find higher values with more massive dust discs ?

Merci pour votre attention
Thank you for your attention
Danke für Ihre Aufmerksamkeit

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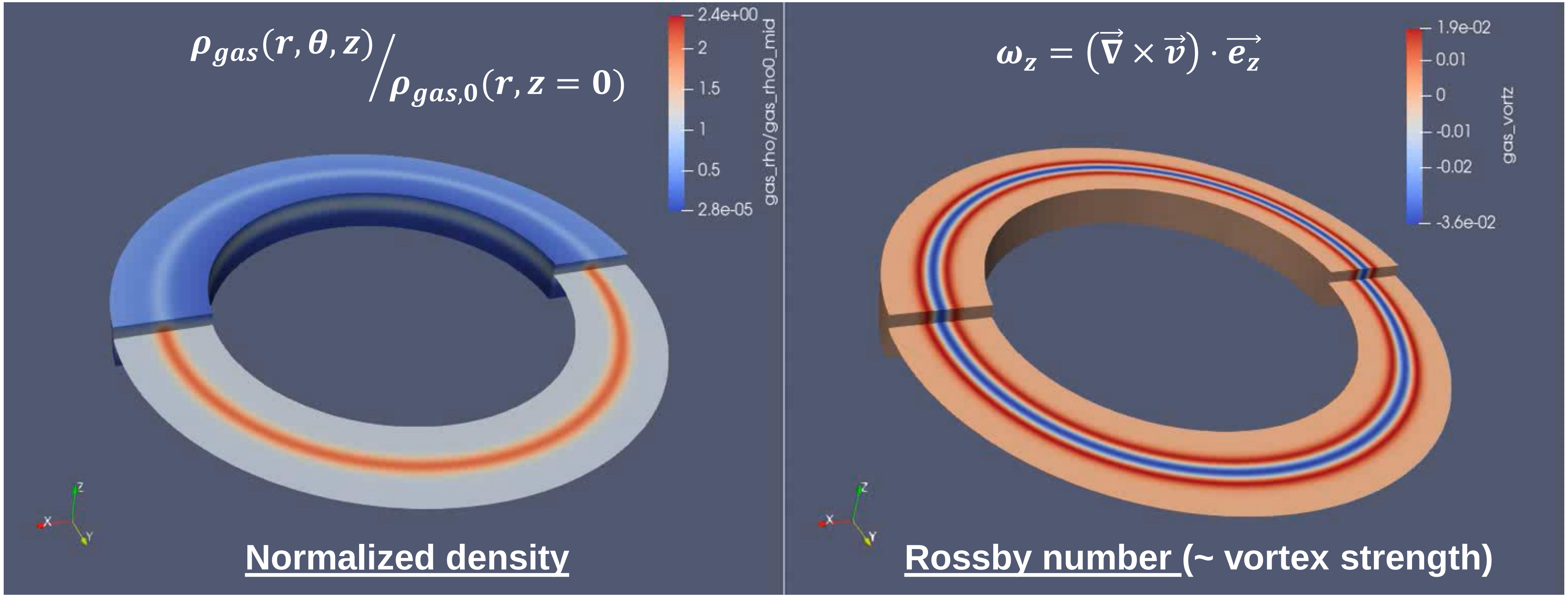
"… Les moyens par lesqueles les hommes arrivent à la connaissance des choses célestes sont à peine moins merveilleux que la nature de ces choses elles-mêmes", Johannes Kepler

How vortices form ?

Example: Rossby Wave Instability (Annular ring is unstable) (Lovelace et al., 1999)



3D simulation performed in Jean Zay cluster with *RoSSBi3D*

Assumptions: Dust embedded in turbulent gaseous environement

If dust SG ignored + gas Gaussian

$$\rho_g = \frac{\Sigma}{\sqrt{2\pi} H_g} \exp\left[-\frac{1}{2}(z/H_g)^2\right]$$

$$\rho_{\rm d} = \rho_{\rm d,mid} \exp\left[-\frac{(\Omega\tau_{\rm s})_{\rm mid}}{\tilde{D}}\left(\exp\left(\frac{Z^2}{2H^2}\right)-1\right)-\frac{Z^2}{2H^2}\right]$$

(Fromang & Nelson 2009)

If dust disc very massive

$$\rho_d = \frac{\Sigma_d}{2Q_d H_d} \mathrm{sech}^2\left(\frac{z}{Q_d H_d}\right)$$

(Klahr & Schreiber 2020, 2021)

Here dust treated separately from gas !

Massive gas disc and constant stopping time with vertical profile

$$\begin{cases} \rho_g(r,z) = \dfrac{\Sigma_g}{2Q_gH_g}\,\mathrm{sech}^2\left(\dfrac{z}{Q_gH_g}\right) \\ \rho_d(r,z) = \dfrac{\Sigma_d}{2Q_gI_1(\xi^2)H_g}\,\mathrm{sech}^2\left(\dfrac{z}{Q_gH_g}\right) \\ \qquad \exp\left[-\xi^2\cosh^2\left(\frac{z}{Q_gH_g}\right)\right] \end{cases}$$

where:

$$I_1(\xi^2) = \frac{\xi^2}{2}\exp\left(-\frac{\xi^2}{2}\right)\left[K_1\left(\frac{\xi^2}{2}\right) - K_0\left(\frac{\xi^2}{2}\right)\right]$$

Does it remind you something ?

This is the stratification found by Fromang & Nelson 2009 when the **disc of gas is massive**.

Not very practical for infering parameters from discs.

Particularly, the scale height of dust is not obvious...