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STUDY OF UNCERTAINTIES IN THE CESAM SOLAR MODEL : PRELIMINARY RESULTS

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Abstract. We present the first results of the study of the inverse propagation of errors. The goal is to obtain the uncertainties for the values of some parameters of the CESAM Solar Model.

1 Introduction

The modeling of stars with a stellar code, e.g. with CESAM, requires the determination of the value of some parameters depending of the value of some observations. Here, we define a *'model'* of the Sun as the union of the set of some given observations and the set of some chosen parameters. Hence, the *'model'* of the Sun with luminosity and surface temperature as observations is different from a *'model'* with luminosity and radius as observations.

The goal of this paper is to determine in a consistent way the uncertainties of the (fitted) values of the parameters depending of the uncertainties of the (observed!) values of the observations. This can be done in inverting the (so-called) *equation of propagation of errors* (see section 2.2).

As illustration, we present a very simple but not so trivial example.

2 Our simple but illustrative model

2.1 Observed values and their uncertainties

For the Solar luminosity, we have $L_{\odot} = (3.8455 \pm 0.0077) \times 10^{33}$ erg/s . The (central) value is taken from "Allen's astrophysical quantities" (Cox 2000, page 340). But for evaluating the related uncertainty, we must use the entry corresponding to the (so-called) "solar constant" (curiously this quantity is quoted with its uncertainty, the Solar luminosity not !).

The effective temperature is taken as $T_{eff} = 5767 \pm 21$ K (Prieto 2001). The value quoted in (Cox 2000, page 341) has no indication about its uncertainty !

We can also found a *value* for the Solar radius (which radius ?, that is the question !) $R_{\odot} = (6.95508 \pm 0.00026) \times 10^{10}$ cm (Cox 2000, page 340).

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2.2 A very little part of formalism

We have the relation :

$$\ln(R_\odot) = 18.590123 + 1/2\ \ln(L_\odot) - 2\ \ln(T_{eff}) \tag{2.1}$$

with the same units as in the previous section.

With luminosity and temperature as parameters, the (transposed) design matrix of our problem is : ${}^tD = \begin{pmatrix} 1 & 0 & 1/2 \\ 0 & 1 & -2 \end{pmatrix}$. The goal is to compute matrices Σ and C (resp. variance-covariance matrix of the observables, of the parameters) which are linked by the relation $\Sigma = D\ C\ {}^tD$, called the *equation of propagation of errors.* Note that C is, in our case, a sub-matrix of Σ and thus the computation ensures the required consistency.

2.3 Determination of a coherent value of the Solar radius

First, the three quoted values in section 2.1 do not verify equation 2.1 (even within their errors). Using this formalism (see section 2.2), we can determine a value of radius (and also its uncertainty). For this, it suffices to introduce any guess value for $R_\odot$ but with a very huge uncertainty (i.e. tells to the program that there is no information about radius), for instance $R_\odot^{guess} = (9.0 \pm 5.0\ 10^{14}) \times 10^{10}$ cm (yes !). We obtain $R_\odot^{comp} = (6.985 \pm 0.051) \times 10^{10}$ cm . It is worth to notice that this value is the same as computed by equation 2.1 and this error is the same as computed with classical (pedestrian) method of evaluating errors.

Now, if we introduce together the three values in the same framework, the overall fitting produces : $L_\odot = (3.84\mathbf{49} \pm 0.0076) \times 10^{33}$ erg/s , $T_{eff} = 57\mathbf{79} \pm \mathbf{3}$ K and $R_\odot = (6.955\mathbf{10} \pm 0.000\mathbf{30}) \times 10^{10}$ cm . Note that the (apparent) very tiny uncertainty of $R_\odot$ leads to a very minor change in radius and transfer the alteration of digits (essentially) on temperature.

But, if we don't trust the quoted value/error of radius (cf. the question : which radius ? of that ?), we can do the same thing as previous but we choose an uncertainty in radius 100 times higher (viz. at 100σ). We obtain a more satisfactory result : $L_\odot = (3.845\mathbf{0} \pm 0.0076) \times 10^{33}$ erg/s , $T_{eff} = 57\mathbf{76} \pm \mathbf{10}$ K and $R_\odot = (6.9\mathbf{61} \pm \mathbf{0.023}) \times 10^{10}$ cm .

3 Conclusion

A more detailed and advanced version of this work is already available (in French) at the following URL : `http://www.obs-nice.fr/pichon/Paris-PNPS.pdf`.

References

Prieto C. A., Lambert D. L. and Asplund M. , ApJ. (2001) L63

Cox A. N. ed., "Allen's astrophysical quantities", 4th ed. (2000) Springer