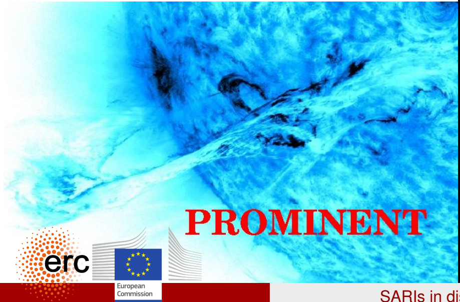
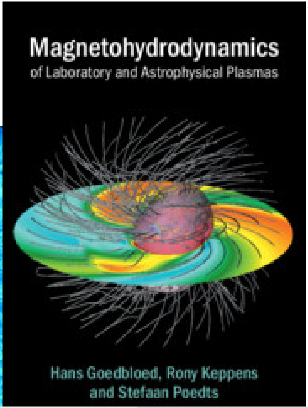
SARIs: A new paradigm for turbulent accretion

Rony Keppens

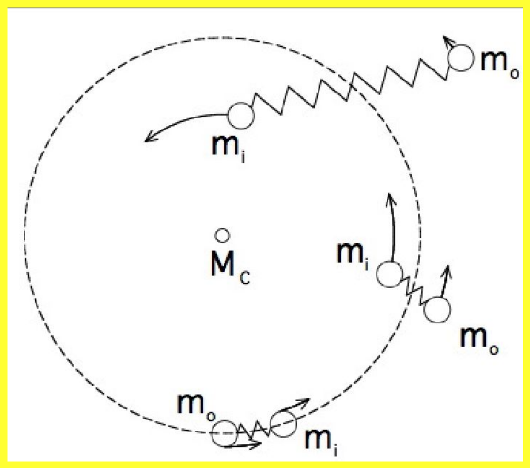
[& Hans Goedbloed]





MRI needs no introduction,
and is invoked in accretion disks of all shapes and sizes

MRI has simple 'mechanical spring' analogy

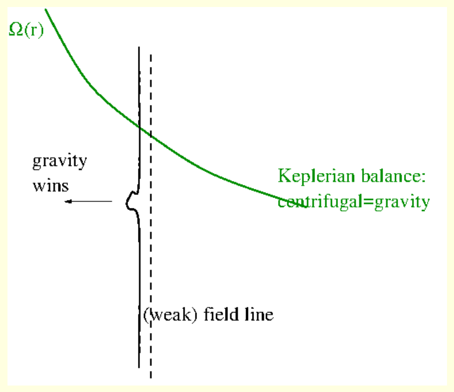


From https://mri.pppl.gov/physics.html

- ingredients: radially decreasing $\Omega(r)$ with r radial distance
  $\Rightarrow$ weak magnetic field (uniform B_z) "acts like a spring"

Lorentz force always $\perp$ **B**, so **more subtle** argument needed

Invoke *enforced isorotation on field line*, then any radially inward displaced element on field line will locally have unbalanced centrifugal/gravitational effects: gravity wins



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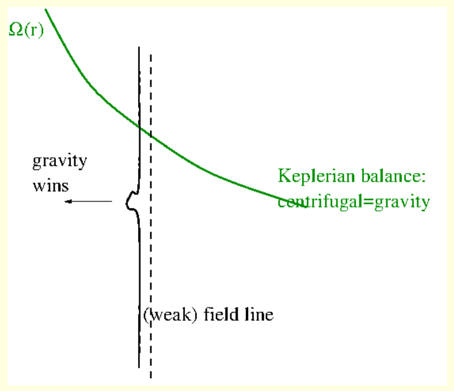

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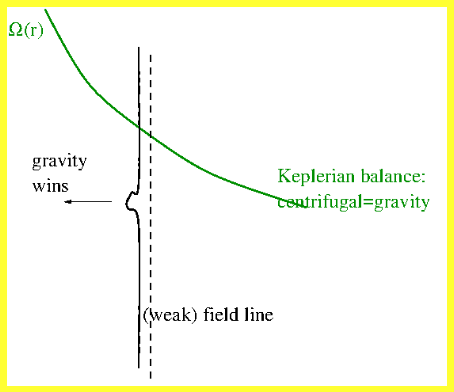

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- derivation starts from ideal MHD, linearize and assume $\exp(-\mathrm{i}\omega t)$, do WKB analysis $\exp(\mathrm{i}k_r r + \mathrm{i}k_z z + \mathrm{i}m\theta)$, and set $m = 0$

  $\Rightarrow$ Balbus & Hawley 1991 (BH91)

1991ApJ...376..214B 1991/07 cited: 3453

A Powerful Local Shear Instability in Weakly Magnetized Disks. I. Linear Analysis

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European Commission

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  - ⇒ axisymmetry obviously excludes dynamo-relevance
  - ⇒ many later results use nonlinear MHD simulations
- current focus in literature
  - ⇒ **protoplanetary disks**: MRI-suppression due to ambipolar diffusion, Hall, . . . , effects on dust redistribution
  - ⇒ **black hole disks**: MRI as primary source to get secondary reconnection, thermal cycles . . .

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- equation of motion in ξ: Lagrangian displacement fluid element

$$\rho \frac{\partial^2 \xi}{\partial t^2} + 2\rho \mathbf{v} \cdot \nabla \frac{\partial \xi}{\partial t} + \underbrace{\nabla \Pi - \mathbf{B} \cdot \nabla \mathbf{Q} - \mathbf{Q} \cdot \nabla \mathbf{B} + \nabla \cdot (\rho \xi) \mathbf{g}}_{\text{Self-adjoint Force operator } -\mathbf{F}(\xi)} - \nabla \cdot [\rho \xi (\mathbf{v} \cdot \nabla) \mathbf{v} - \rho \mathbf{v}\mathbf{v} \cdot \nabla \xi] = 0,$$

$\Rightarrow$ Π Eulerian perturbation of the total pressure,

$\Rightarrow$ **Q** Eulerian perturbation of magnetic field.

- **given stationary v, gravitating g, magnetized B equilibrium (1D, 2D, 3D): governs eigenmodes** $\xi(t) \propto \exp(-i\omega t)$
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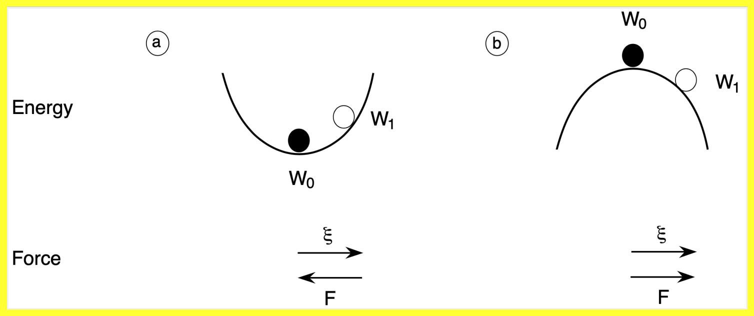
Recap on static ($\mathbf{v} = \mathbf{0}$) case!

- **no flow:** $\mathbf{F}(\boldsymbol{\xi}) = -\rho\omega^2\boldsymbol{\xi}$, always real ω^2 [Bernstein et al 1958]

  $\Rightarrow$ clear physics: $\omega^2 > 0$ has force opposing $\boldsymbol{\xi}$ $\Rightarrow$ stable!



  $\Rightarrow$ $-\frac{1}{2}\int \boldsymbol{\xi}^* \cdot \mathbf{F}(\boldsymbol{\xi})\, dV$ quantifies plasma potential energy

Stationary case

- when $\xi(t) \propto \exp(-i\omega t)$ and moving $\mathbf{v} \neq \mathbf{0}$ background

$$\rho \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} \equiv -\rho\omega^2 \boldsymbol{\xi} = \mathbf{G}(\boldsymbol{\xi}) - 2\omega U \boldsymbol{\xi}$$

$\Rightarrow$ **G** generalized operator $\mathbf{F}(\boldsymbol{\xi}) + \nabla \cdot [\rho\boldsymbol{\xi}(\mathbf{v}\cdot\nabla)\mathbf{v} - \rho\mathbf{v}\mathbf{v}\cdot\nabla\boldsymbol{\xi}]$

$\Rightarrow$ **$\mathbf{G}(\boldsymbol{\xi})$ self-adjoint**, i.e. $\int \boldsymbol{\eta}^* \cdot \mathbf{G}(\boldsymbol{\xi})\, dV = \int \boldsymbol{\xi} \cdot \mathbf{G}(\boldsymbol{\eta}^*)\, dV$

$\Rightarrow$ Doppler-Coriolis operator from $2\rho\mathbf{v}\cdot\nabla\frac{\partial\boldsymbol{\xi}}{\partial t} \equiv 2\omega U\boldsymbol{\xi}$

$\Rightarrow$ ***U* also self-adjoint!** where $U \equiv -\mathrm{i}\rho\mathbf{v}\cdot\nabla$

- possibly **intrinsically complex** eigenvalues $\omega \equiv \sigma + i\,\nu$

$\Rightarrow$ shear flow drives e.g. **Kelvin-Helmholtz instabilities**

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- normal modes $\exp(-\mathrm{i}\omega t)$ obey *spectral equation*

$$-\rho\omega^2\boldsymbol{\xi} = \mathbf{G}(\boldsymbol{\xi}) - 2\omega U\boldsymbol{\xi}$$

$\Rightarrow$ quadratic eigenvalue problem (supplement with BCs), now **governed by two-self-adjoint operators!**

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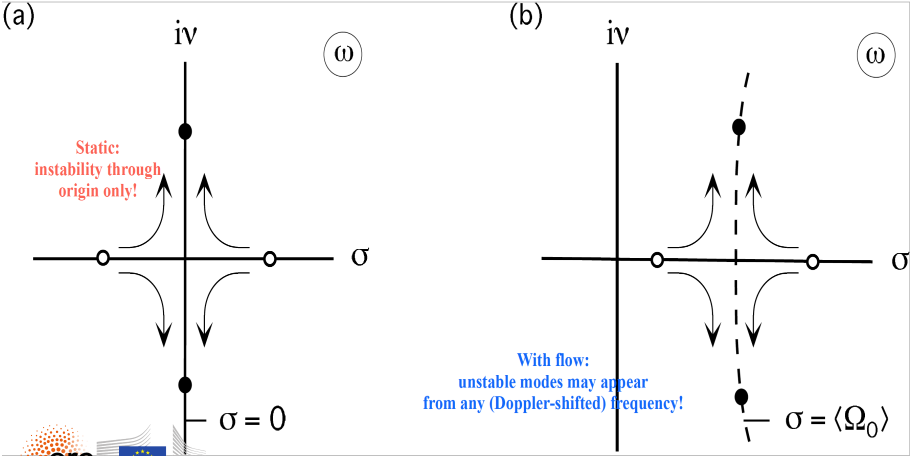
$$-\rho\omega^2 \xi = \mathbf{G}(\xi) - 2\omega U \xi$$

- static ⇒ flow: instability can be away from marginal frequency!



- **if eigenvalue-eigenvector $\omega - \xi$ known,** from

$$-\rho\omega^2\boldsymbol{\xi} = \mathbf{G}(\boldsymbol{\xi}) - 2\omega U\boldsymbol{\xi}$$

$\Rightarrow$ introduce (normed) versions of $V \equiv \frac{1}{2}\int \boldsymbol{\xi}^* \cdot U\boldsymbol{\xi}\, dV$ and $W = -\frac{1}{2}\int \boldsymbol{\xi}^* \cdot \mathbf{G}(\boldsymbol{\xi})\, dV$, get quadratic $\omega^2 - 2\omega\bar{V} - \bar{W} = 0$

$\Rightarrow$ **holds for actual (possibly complex) eigenvalues, for which quadratic forms V, W are real!**

$\Rightarrow$ **Problem Solved!**, *not really...*

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- specify to "cylindrical disk": consider completely general equilibrium obeying MHD force balance in disk equatorial plane

$$\left(p + \frac{B_\theta^2 + B_z^2}{2}\right)' = \rho \underbrace{\left(\frac{v_\theta^2}{r} - \frac{GM_*}{r^2}\right)}_{\text{Keplerian flow}} - \frac{B_\theta^2}{r}$$

$\Rightarrow$ then governing linear ideal MHD equations for perturbations $\delta f(r,z,\theta,t) = \hat{f}(r)\exp(\mathrm{i}k_z z + \mathrm{i}m\theta - \mathrm{i}\omega t)$ obey second order ODE in $\chi = r\xi_r$

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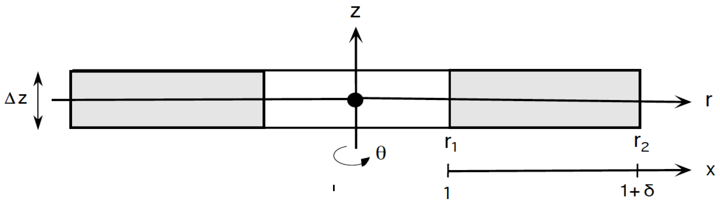
- **math 101**: 2nd order ODE needs 2 BCs!

$$\frac{d}{dr}\left(\frac{N}{D}\frac{d\chi}{dr}\right)+\left[A+\frac{B}{D}+\left(\frac{C}{D}\right)'\right]\chi=0 \qquad \chi(r_1)=0 \;(\text{left}), \qquad \chi(r_2)=0 \;(\text{right}).$$



- ω ranges where $N(r,\omega)=0$ are special: Singularities!
  $\Rightarrow$ forward and backward Alfvén and slow continua (real!)

$$\Omega^{\pm}_{A,S}=\underbrace{\frac{m v_\theta(r)}{r}}_{\text{Doppler shift }\ \Omega_0(r)}\pm\omega_{A,S}(r)$$

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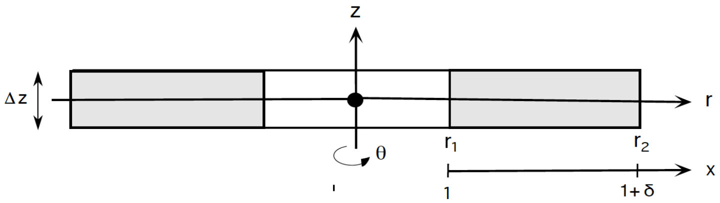

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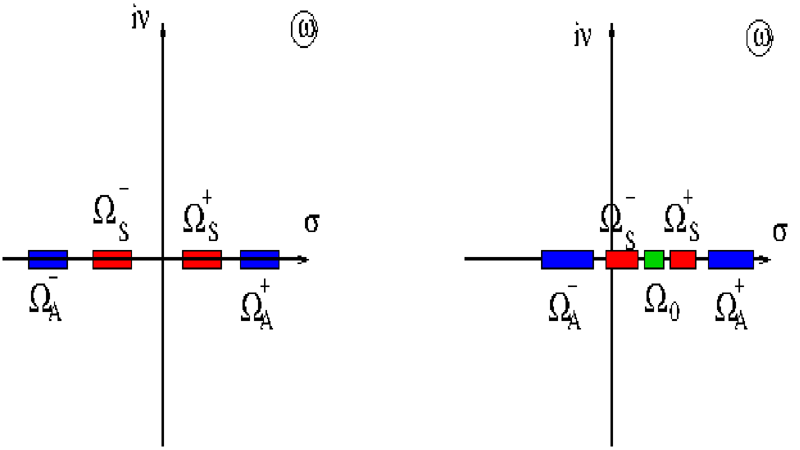
schematic locations of continua in complex ω plane



Static: left-right+up-down symmetry   Stationary: up-down symmetry

- loci of other, discrete eigenvalues in complex plane?

  $\Rightarrow$ pick any value $\omega \equiv \sigma + i\,\nu$ in plane, start at one boundary of domain, integrate (complex) ODEs throughout the domain to other boundary (can 'always' be done accurately)

  $\Rightarrow$ quantify values obtained for $\bar{V}$ and $\bar{W}$

  $\Rightarrow$ $\bar{V}$ turns out always real, for any complex ω, BCs irrelevant!

  $\Rightarrow$ eigenvalues satisfy all BCs, and make $\mathrm{Im}(\bar{W}) = 0$!

  $\Rightarrow$ just plot curve(s) $\mathrm{Im}(\bar{W}) = 0$: **solution path**

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- physical meaning of a complex $\bar{W}$?

$$W = -\frac{1}{2}\int \xi^* \cdot \mathbf{G}(\xi)\, dV = \underbrace{W^p}_{\text{plasma energy (real)}} + W_{\text{com}}$$

where the **complex complimentary energy** is

$$W_{\text{com}}[\xi] \equiv \frac{1}{2}\int \xi_n^* \Pi(\xi)\, dS$$

$\Rightarrow$ **added energy needed to ensure a perfect resonance** with complex frequency ω (zero for eigenmode!)

$\Rightarrow$ opens up one boundary!

$\Rightarrow$ surface integral (i.e. a quantity evaluated at one radius for cylindrical disk) that evaluates local total pressure/displacement.

- Goedbloed 2018a,b introduced **'Spectral Web' method**
  1. for radial range $r \in [r_1, r_2]$, and arbitrary complex ω (not in real continua), use 2nd order ODE in $\xi \equiv \xi_r$
  2. start from left BC, integrate to internal $r_{\rm mix}$
  3. start from right BC, integrate to same $r_{\rm mix}$
  4. can always exploit freedom in amplitude (linear problem) to make ξ continuous at $r_{\rm mix}$
  5. almost always, the derivate ξ' jumps and this jump (in Π) locally quantifies $W_{\rm com}$
- only needs **basic plotting routines** to visualize

$$Im(W_{\rm com}) = 0 \quad i.e. \quad the \;\; solution \;\; path$$

$$Re(W_{\rm com}) = 0 \quad i.e. \quad the \;\; conjugate \;\; path$$

$\Rightarrow$ while the choice of $r_{\rm mix}$ influences the shape/location of these curves, their crossings are unaltered, since they are perfect resonances: actual eigenmodes!

In summary:

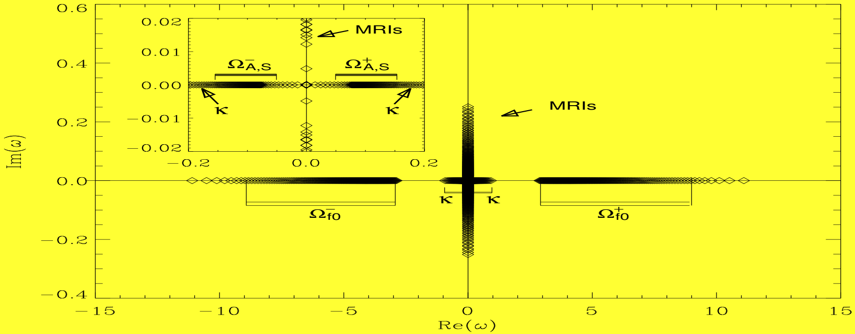
- using self-adjointness of both occuring operators, for given equilibrium and mode numbers (m, k_z)

  $\Rightarrow$ possible to localize all eigenmodes, at intersections of (easily) computable curves in the complex frequency plane

  $\Rightarrow$ the curve intersections have the physical meaning of locating perfect resonances!

Seismology of accretion disks [Keppens et al, *ApJL* **569**, 2002]

- $\beta = 2000$, helical **B**, axisymmetric modes $\Rightarrow$ Doppler $\mathbf{k} \cdot \mathbf{v} = 0$

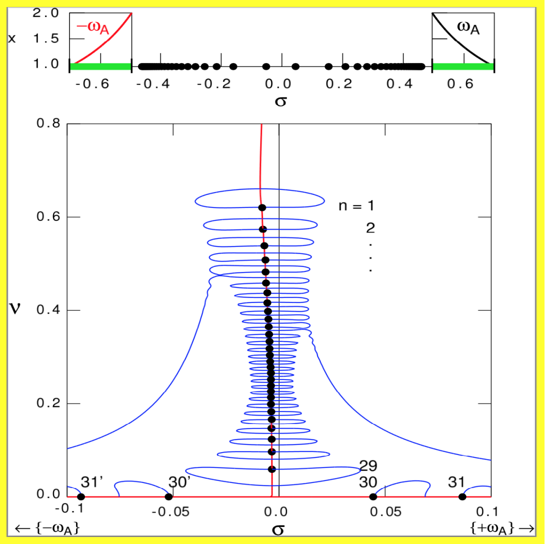


- backward & forward fast $F^{\pm}$, Alfvén $A^{\pm}$, slow $S^{\pm}$
- HD epicyclic modes, frequency $\kappa^2 \equiv 2v_{\theta,0}(rv_{\theta,0})'/r^2$

Magneto-rotational instability in slow subspectrum

Weakly magnetized disks: spectral web

- zoom on MRIs ($m = 0$)
  - $\Rightarrow$ far from $\Omega^{\pm}_{A,S}$!
  - $\Rightarrow$ ok for WKB!



- **only finite # discrete unstable MRIs**!
  - $\Rightarrow$ WKB $\chi(r) = p(r) \exp\left[\pm \mathrm{i} \int q(r)\, dr\right]$ gives

$$(k^2 + q^2)(\omega^2 - \omega_A^2)^2 - k^2 \kappa_e^2 (\omega^2 - \omega_A^2) - 4k^2 \Omega^2 \omega_A^2 = 0\,,$$

  - $\Rightarrow$ part of infinite sequence of mostly stable modes

$$\omega \approx \pm[\omega_{A2} + \delta\omega'_A \exp(-n\pi/p)]$$

- WKB **misses overstable aspect** of modes (assumes real χ, Π)
  - $\Rightarrow$ and avoids the BCs!

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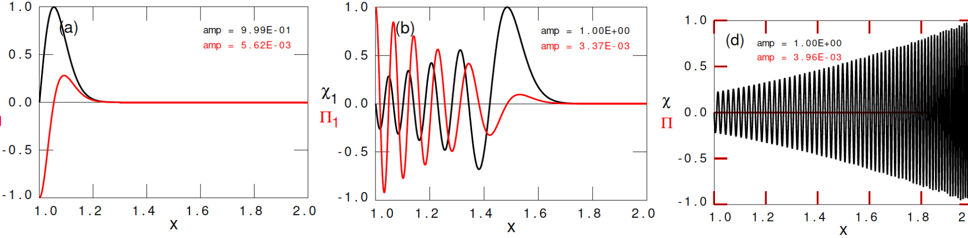
Showing $n = 1$ MRI, $n = 20$ MRI and stable $n = 130$



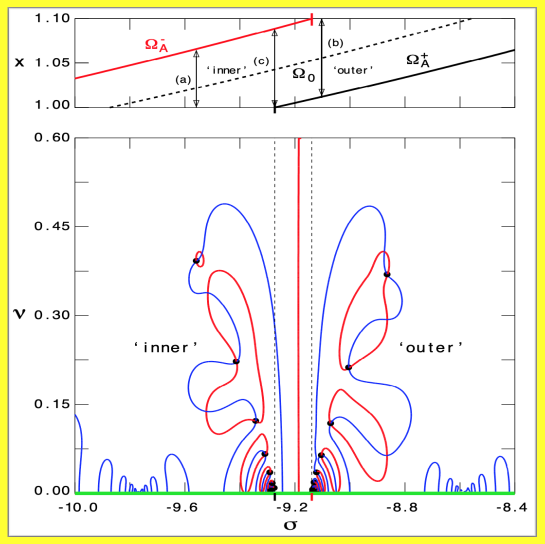
- **MRI is GLOBAL, sensitive to BCs**
- radially localized modes are STABLE

Weakly magnetized disks: SARIs

SARIs ($m = -10, k = 70$)

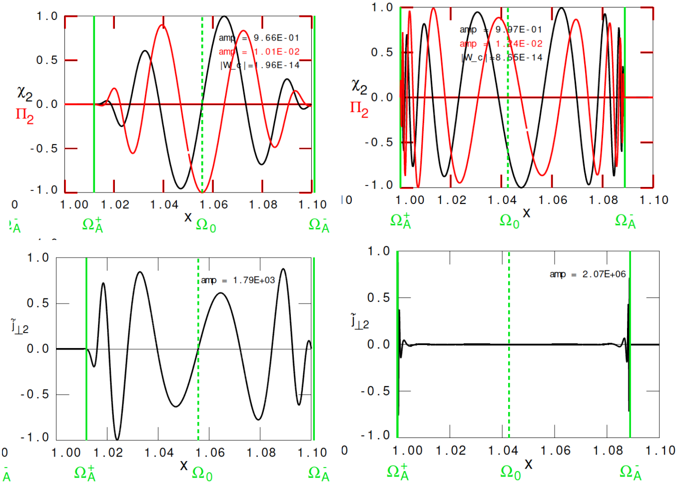
$\Rightarrow$ overlap $\Omega_A^{\pm}$!

$\Rightarrow$ not ok for WKB!



- non-axisymmetric modes on thin radial slice $\delta = 0.1$
  - $\Rightarrow$ show **2 infinite sequences of ALL UNSTABLE** modes
  - $\Rightarrow$ inner vs. outer, co- vs. counter-rotating SARIs

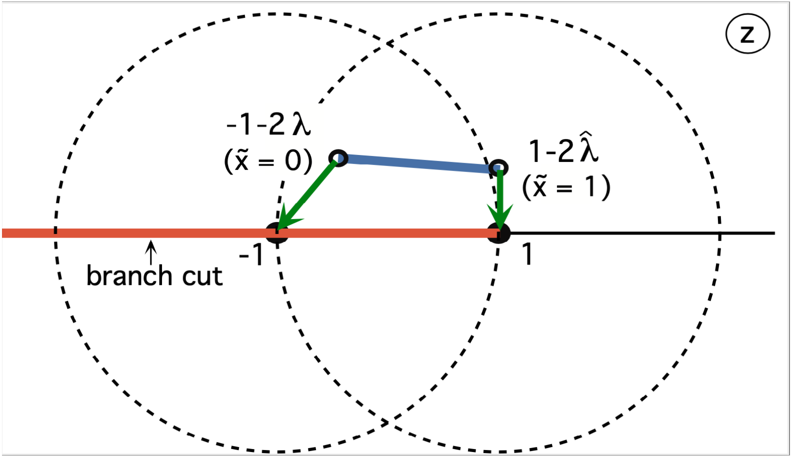
Showing 4th outer SARI, 10th inner SARI



- **SARI is insensitive to one BC**

$\Rightarrow$ creates current distribution acting as virtual wall!

I now skip over a true mathematical 'tour-de-force' (by Hans) on how to get dispersion relation for these modes as solutions of a Legendre equation, where two near-singularities must be handled properly!!!

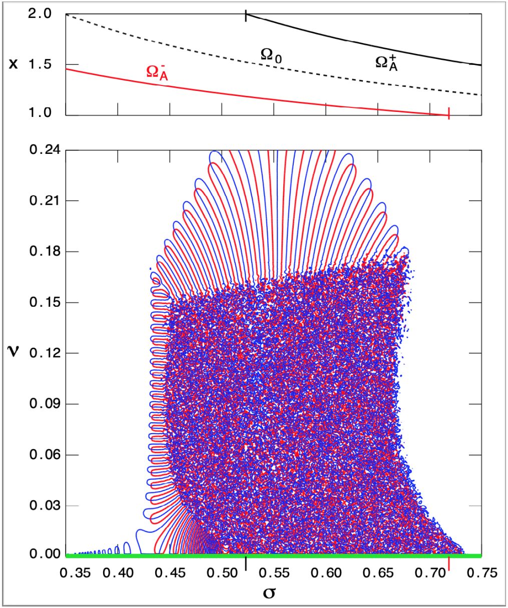


There is perfect agreement in theory/numerics!

- MRI is $m = 0$
  - $\Rightarrow$ is global
  - $\Rightarrow$ finite # unstable
  - $\Rightarrow$ sensitive to 2 BCs
  - $\Rightarrow$ WKB-amenable

- SARIs are $m \neq 0$
  - $\Rightarrow$ global to local
  - $\Rightarrow$ infinite # unstable
  - $\Rightarrow$ sensitive to 1 BC
  - $\Rightarrow$ needs to treat two near-singularities
  - $\Rightarrow$ need $\underbrace{m\Omega \gg \omega_A}_{SARI}$

- for SARIs, took thin radial slice $\delta = 0.1$, let's consider larger δ



- Spectral web 'fragments'? **What the heck?**

  ⇒ further zoom in? **does not help**

  ⇒ Other spectral solver (*Legolas*, see Claes et al, 2020, ApJS or `http://legolas.science`)? **inconclusive in terms of converged spectral results**

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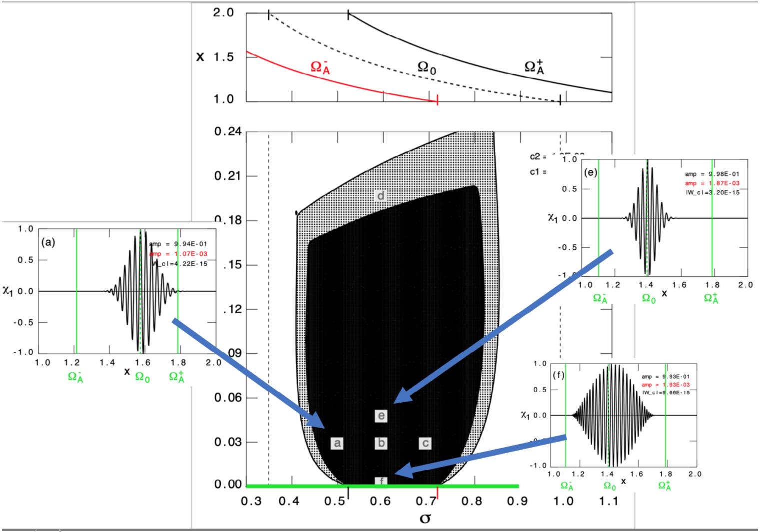
- Remember physical meaning of spectral web: visualizing W_{com}

  $\Rightarrow$ there are finite 2D regions in ω-plane where W_{com} is tiny

  $\Rightarrow$ since W_{com} non-zero: NO eigenmodes!!!!

  $\Rightarrow$ but you just need a minute addition of energy!

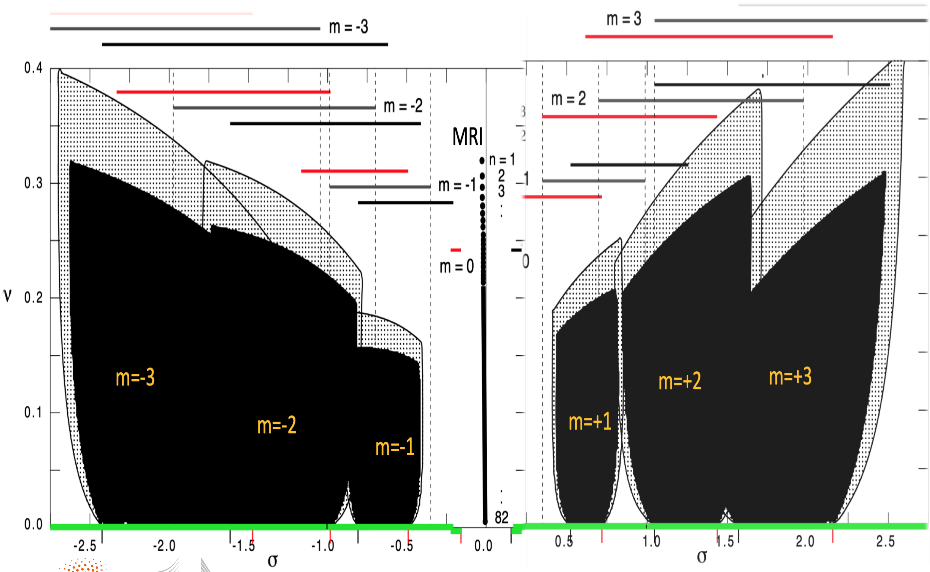
- allow deviations from eigenmodes at MACHINE PRECISION!



- peculiar behavior fully understood from complex analysis!
- we have now quasi-modes, found over 2D region in ω
  - $\Rightarrow$ easily excited
  - $\Rightarrow$ insensitive to BOTH BCs
  - $\Rightarrow$ 3D localized ($m \neq 0$, large k_z, finite wave package in r)
  - $\Rightarrow$ **what more do you need for turbulence?**

- http://arxiv.org/abs/2201.11551 or **2022, ApJ Supplement Series 259, 65** conjectures a **radically NEW paradigm** for ideal MHD instability in (weakly) magnetized disks

  ⇒ "We conjecture that the onset of turbulence in accretion disks is governed, not by the excitation of discrete axisymmetric Magneto-Rotational Instabilities, but by the **excitation of modes from these two-dimensional continua of quasi-discrete non-axisymmetric Super-Alfvénic Rotational Instabilities.**"


m = -3
m = -2
m = -1
m = 0
MRI
n = 1
2
3
82
m = 3
m = 2
m=-3
m=-2
m=-1
m=+1
m=+2
m=+3
ν
σ
0.4
0.3
0.2
0.1
0.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5

Take-Home

- MRI is relevant for disks, but . . .

  ⇒ SARIs are more relevant! (dynamo)

- look beyond pure eigenmodes: enter quasi-continuum SARIs!

  "singularity is almost invariably a clue"

  (Bender & Orszag on Arthur Conan Doyle's Sherlock Holmes)

- **Very satisfying to find out that black holes and their disks thrive on singularities!**

Take-Home

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  - $\Rightarrow$ SARIs are more relevant! (dynamo)
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- not all stella!